Zombie Apocalypse: An Epidemic Model

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Overview

• Our interest
• Zombie basics
• Epidemic (SIR) model
• Munz et al., 2009
• Modifications
Chapter 4

**WHEN ZOMBIES ATTACK!: MATHEMATICAL MODELLING OF AN OUTBREAK OF ZOMBIE INFECTION**

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What is a zombie?

• “Undead”
• Eat human flesh
• Infect healthy
• Difficult to kill (destroy brain)

Photo credit: thewalkingdeadstrream.net
What is a zombie?

Main dynamics:
1. How you become a zombie
   Sick, Bitten, Die

2. How you get rid of zombies
   Cure, Death
Some properties of a simple stochastic epidemic model of SIR type

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Epidemic Model

Susceptible: $S$  +  Infected: $I$  +  Recovered: $R$  =  Total Pop

Photo credit: http://whatthehealthmag.wordpress.com
(change in quantity S) = -(constant)
(change in quantity $S$) = $-(\text{constant})$

$S' = -\theta$
\( \text{(change in quantity } S \text{)} = -(\text{constant}) \)

\[ S' = -\theta \]

\[ l' = +\theta \]
Epidemic Model

\[ I' = - \zeta I \]
\[ R' = +\zeta I \]
Epidemic Model

\[ S' = -\beta SI \]
\[ I' = +\beta SI \]
Epidemic Model

\[ S' = -\beta SI \]
\[ I' = +\beta SI - \zeta I \]
\[ R' = +\zeta I \]
Epidemic Model

Example of SIR dynamics for influenza

Sebastian Bonhoeffer, SIR models of epidemics
Epidemic Model

Modifications

• Death rates
• Latent periods (SEIS)
• Ability to recover (SEIR)
• Ability to become susceptible again (SIRS)
When Zombies Attack!: Mathematical Modelling of an Outbreak Zombie Infection
Munz, Hudea, Imad, Smith (2009)

Goals:
• Model a zombie attack, using biological assumptions based on popular zombie movies
• Determine equilibria and their stability
• Illustrate the outcome with numerical solutions
• Introduce epidemic modeling with fun example
Conclusions:

• Disastrous outbreak unless aggressive tactics
• Collapse of civilization
“When Zombies Attack!: Mathematical Modelling of an Outbreak Zombie Infection”
Munz, Hudea, Imad, Smith (2009)

Great idea, but…

• Models don’t match any film
• All results depend on poor model assumptions
• No data is used
• Wrong parameters given
Munz Model

\[ S \xrightarrow{\beta SZ} Z \quad \xleftarrow{\zeta R} R \quad \alpha SZ \]

- **Susceptible**: \( S' = -\beta SZ \)
- **Zombie**: \( Z' = \beta SZ + \zeta R \)
- **Removed**: \( R' = -\zeta R \)
Munz Model

If zombies are killed, they are soon recycled into the zombie population

Zombies never die
Munz Model

We can find no movies where the $\zeta R$ term is a reasonable assumption.
Reproduce results

• Before starting our models, replicate theirs
• Parameters are drastically wrong

Based on paper:                     Based on model:
\( \beta = .0095 \)               \( \beta = .0028 \)
\( \rho = .005 \)                 \( \rho = 5 \)
\( \zeta = .0001 \)              \( \zeta = 5 \)
Latent Outbreak

Munz paper

Munz equations and parameters
Latent Outbreak

Munz paper

Munz equations, but our fixed parameters
Quarantine

Munz paper

Munz equations and parameters
Quarantine

Munz paper

Munz equations, but our fixed parameters
Upgrades

Shaun of the Dead
Walking Dead
Zombieland
28 Days Later
Resident Evil
Upgrades

Night of the Living Dead
Dawn of the Dead
Day of the Dead
Data Collection

• Watch films, pause to take population count whenever zombies are in the scene
• Record time (within film’s world) versus population increase
Shaun of the Dead

Data points based off film

Slope = 0.404
Intercept = -7.964
Shaun of the Dead

\[ S' = -\beta SZ \]
\[ Z' = \beta SZ - \alpha SZ \]
\[ X' = + \alpha SZ \]
Night of the Living Dead

Slope = 0.595

Intercept = -0.082
Night of the Living Dead

\[ S' = -\beta SZ \]
\[ Z' = \zeta R - \alpha SZ \]
\[ R' = \beta SZ - \zeta R \]
\[ X' = + \alpha SZ \]
Night of the Living Dead
50000 Simulations

\[ \hat{\alpha} = 0.0342 \pm 0.00485 \pm [1\sigma] \]

\[ Z \xrightarrow{\alpha} X \]

\[ \hat{\beta} = 0.0445 \pm 0.00577 \pm [1\sigma] \]

\[ S \xrightarrow{\beta} R \]
Joint Distribution
Conclusions

• Zombie infection would likely be disastrous, but not inevitable as Munz et al. (2009) suggestions

• Data are necessary to make reasonable models and parameter estimations