Truth, Beauty, and BCM Oscillations: New Results

Outline

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  - Assumptions and Calculations
  - Interpretation of Results
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  - Assumptions and Calculations
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Simple Oscillatory Behavior

$$\begin{aligned} c &= md \\ \dot{m} &= \eta c(c-\theta)d \\ \theta &= \frac{1}{\tau}\int_{-\infty}^{t} c^2(t')e^{-(t-t')/\tau}dt' \end{aligned}$$

and assume

$$m = m_o + m_1 \sin\omega t$$

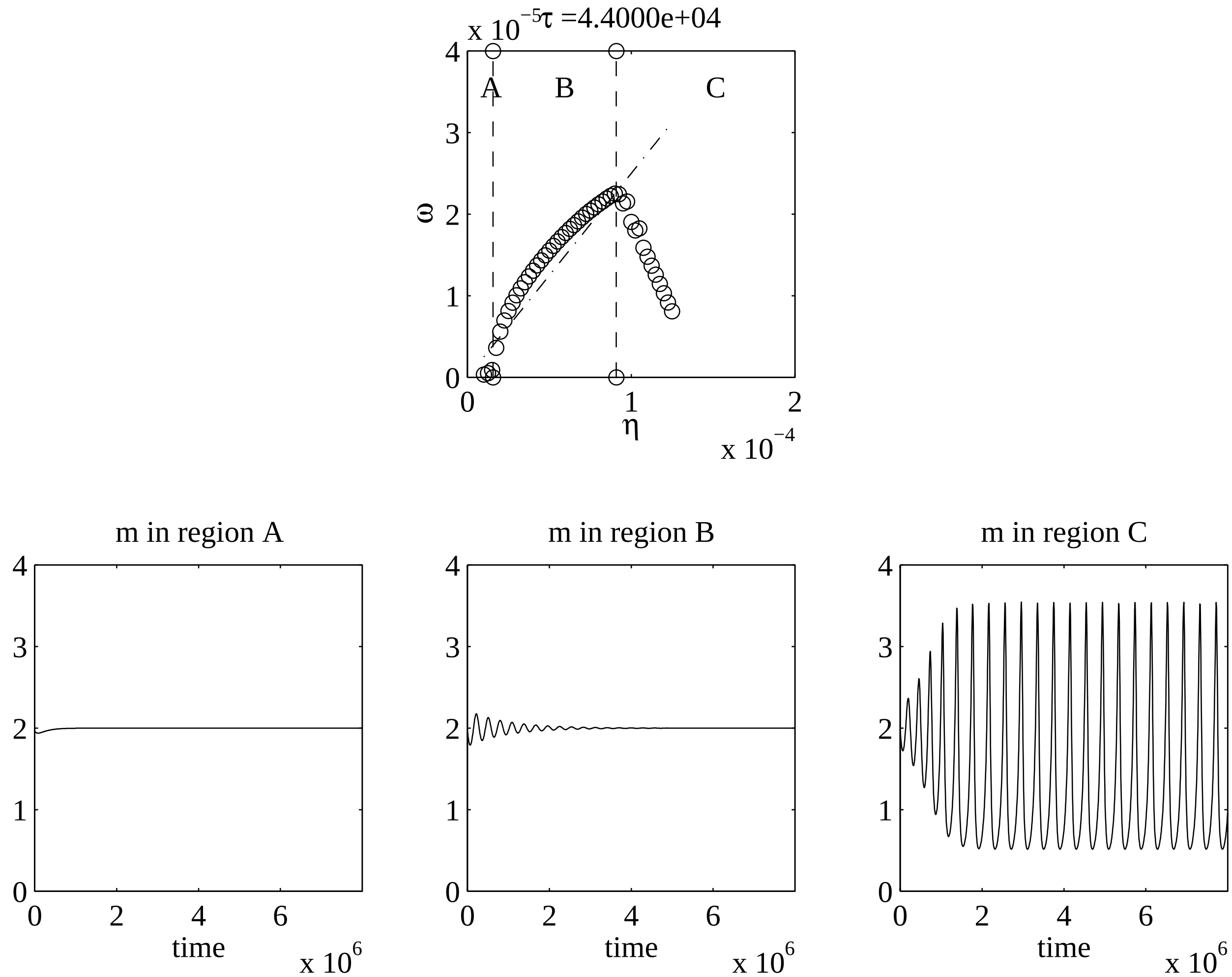
where $m_1 \ll 1$ and $m_o = 1/d$ (fixed point).

$$\dot{m} - \eta c(c-\theta)d = 0 = \frac{A_1}{B}\sin(\omega t) + \frac{A_2}{B}\cos(\omega t)$$

$$\begin{aligned} A_1 &= m_1\eta d^2(1-\omega^2\tau^2) = 0 \\ A_2 &= m_1\omega(1+\omega^2\tau^2 - 2\eta d^2\tau) = 0 \\ B &= 1+\omega^2\tau^2 \neq 0 \end{aligned}$$

$$\begin{aligned} \omega &= \pm\frac{1}{\tau} \\ \frac{1}{\tau} &= d^2\eta \end{aligned}$$

Simple Oscillations: Example



Recipe for Simulations

- Choose $m_{\text{initial}} = 1/(d + 0.2d)$ and $\theta_{\text{initial}} = 1.1$ to get close to fixed point
- Choose range of η, given τ, to include all three behavior regions
- Iterate learning rule for $\approx 5 \cdot 10^6$ iterations
- Fit to damped sine, using Fourier tranform to aid in initial coefficient guesses

Damped Oscillatory Behavior

$$
\begin{aligned}
c &= md \\
\dot{m} &= \eta c(c-\theta)d \\
\theta &= \frac{1}{\tau}\int_0^t c^2(t')e^{-(t-t')/\tau}dt' + \theta_o
\end{aligned}
$$

and assume

$$m = m_o + m_1 e^{i\omega t}e^{-gt}$$

where $m_1 \ll 1$ and $m_o = 1/d$ (fixed point).

$$
\begin{aligned}
\dot{m} - \eta c(c-\theta)d &= 0 \\
0 &= \left\{\frac{A_1}{B}e^{-gt}\sin(\omega t) + \cdots\right\} + \\
&\quad i\left\{\frac{-A_1}{B}e^{-gt}\cos(\omega t) + \cdots\right\}
\end{aligned}
$$

Damped Oscillatory Behavior: Calculations

$$
\begin{aligned}
0 \;=\; & \left\{\frac{A_1}{B}e^{-gt}\sin(\omega t) + \frac{A_2 + A_3}{B}e^{-gt}\cos(\omega t) + \right. \\
& \left. A_4 e^{-t/\tau}e^{-gt}\cos(\omega t) + \frac{A_5}{B}e^{-t/\tau} + A_6\right\} + \\
& i\left\{\frac{-A_1}{B}e^{-gt}\cos(\omega t) + \frac{A_2 + A_3}{B}e^{-gt}\sin(\omega t) + \right. \\
& \left. A_4 e^{-t/\tau}e^{-gt}\sin(\omega t) + \frac{-2A_4\omega\tau}{B}e^{-t/\tau}\right\}
\end{aligned}
$$

$$
\begin{aligned}
A_1 \;&=\; \omega m_1(-\tau^2\omega^2 - g^2\tau^2 + 2g\tau - 1 + 2\eta\tau d^2) = 0 \\
A_2 \;&=\; m_1(-\tau^2\omega^2 g - \tau^2\omega^2 d^2\eta + \eta d^2 - \eta d^2 g^2\tau^2 - g^3\tau^2 + \\
&\quad\; 2g^2\tau - g) \\
A_3 \;&=\; m_1\theta_o(-2\eta d^2 g\tau + \tau^2\omega^2 d^2\eta + \eta d^2 g^2\tau^2 + \eta d^2) \\
A_2 \;&+\; A_3 = 0 \\
A_4 \;&=\; -d^2\eta m_1 = 0 \\
A_5 \;&=\; \eta d(2m_1 dg\tau + 2g\tau - g^2\tau^2 - \tau^2\omega^2 - 1 - 2m_1 d) = 0 \\
A_6 \;&=\; \theta_o\eta d = 0 \\
B \;&=\; \tau^2\omega^2 + g^2\tau^2 - 2g\tau + 1 \neq 0
\end{aligned}
$$

Damped Oscillatory Behavior: Result

$$
\begin{aligned}
\theta_o &= 0 \\
m_1 &= -2\frac{\tau\eta d}{1+\tau\eta d^2} \\
\omega &= \pm\frac{1}{2}\frac{\sqrt{6\tau\eta d^2 - 1 - (\tau\eta d^2)^2}}{\tau} \\
g &= \frac{1}{2}\frac{(1-\tau\eta d^2)}{\tau}
\end{aligned}
$$

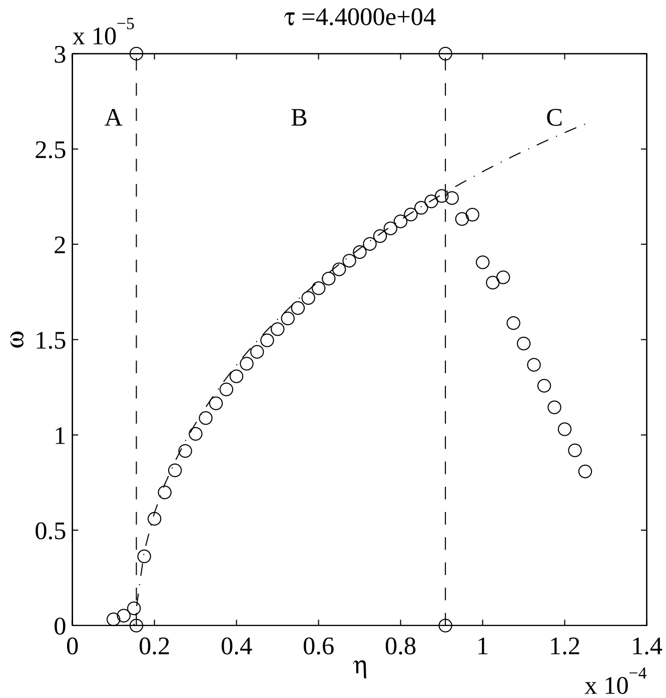
Regions in the Parameter Space

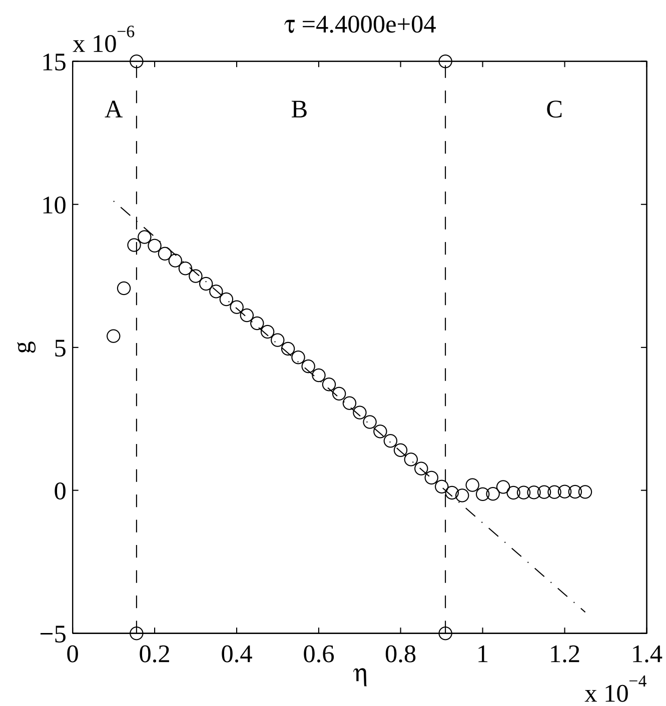
$$\alpha \equiv \tau \eta d^2$$

$$\omega = \pm \frac{1}{2} \frac{\sqrt{6\alpha - 1 - \alpha^2}}{\tau}$$
$$g = \frac{1}{2} \frac{(1 - \alpha)}{\tau}$$

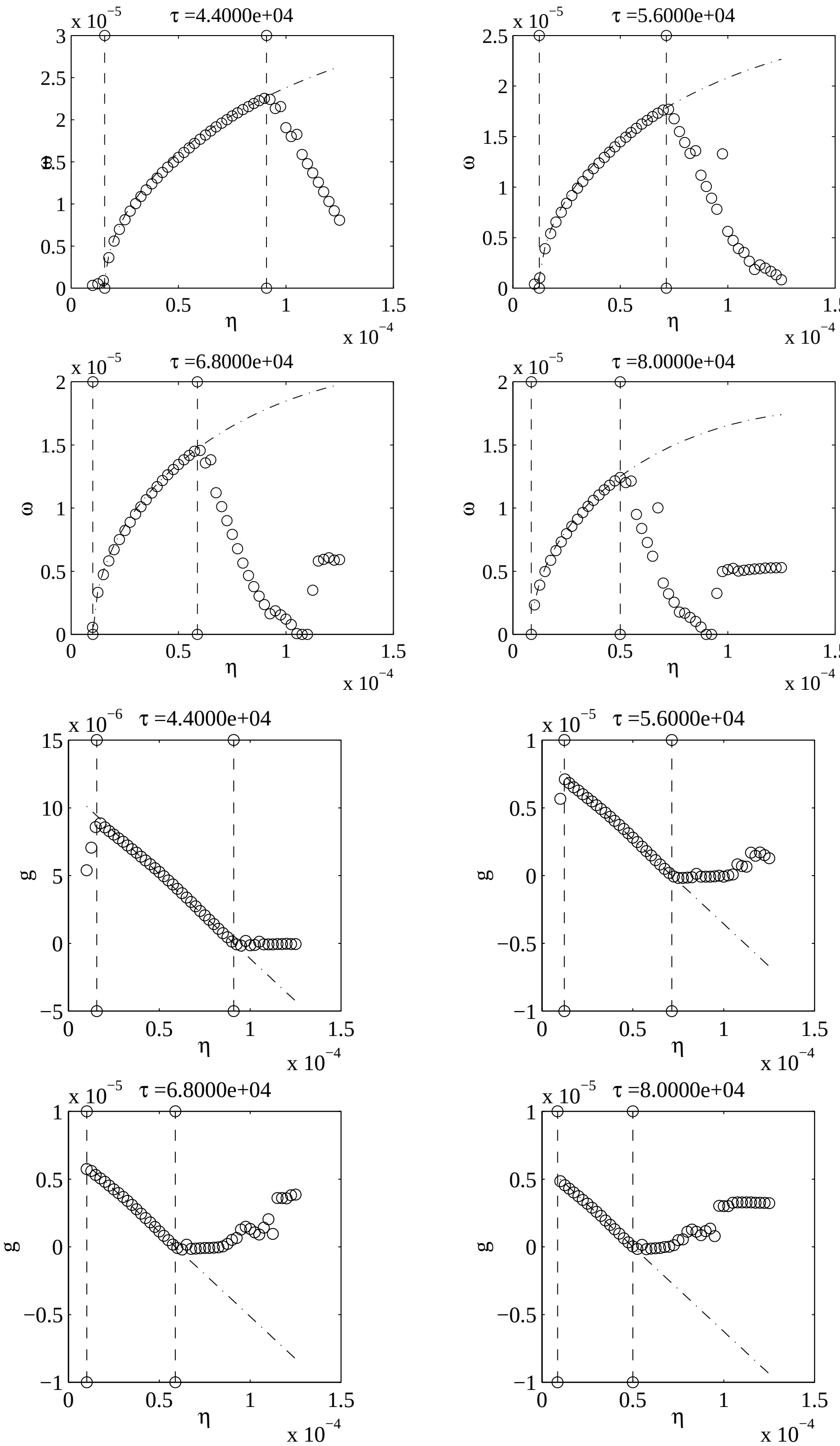
- **Region A:** $6\alpha - 1 - \alpha^2 \leq 0$
- **Region B:** $6\alpha - 1 - \alpha^2 > 0$ and $\alpha < 1$ $(g > 0)$
- **Region C:** $\alpha > 1$ $(g < 0)$

Damped Oscillations: Example

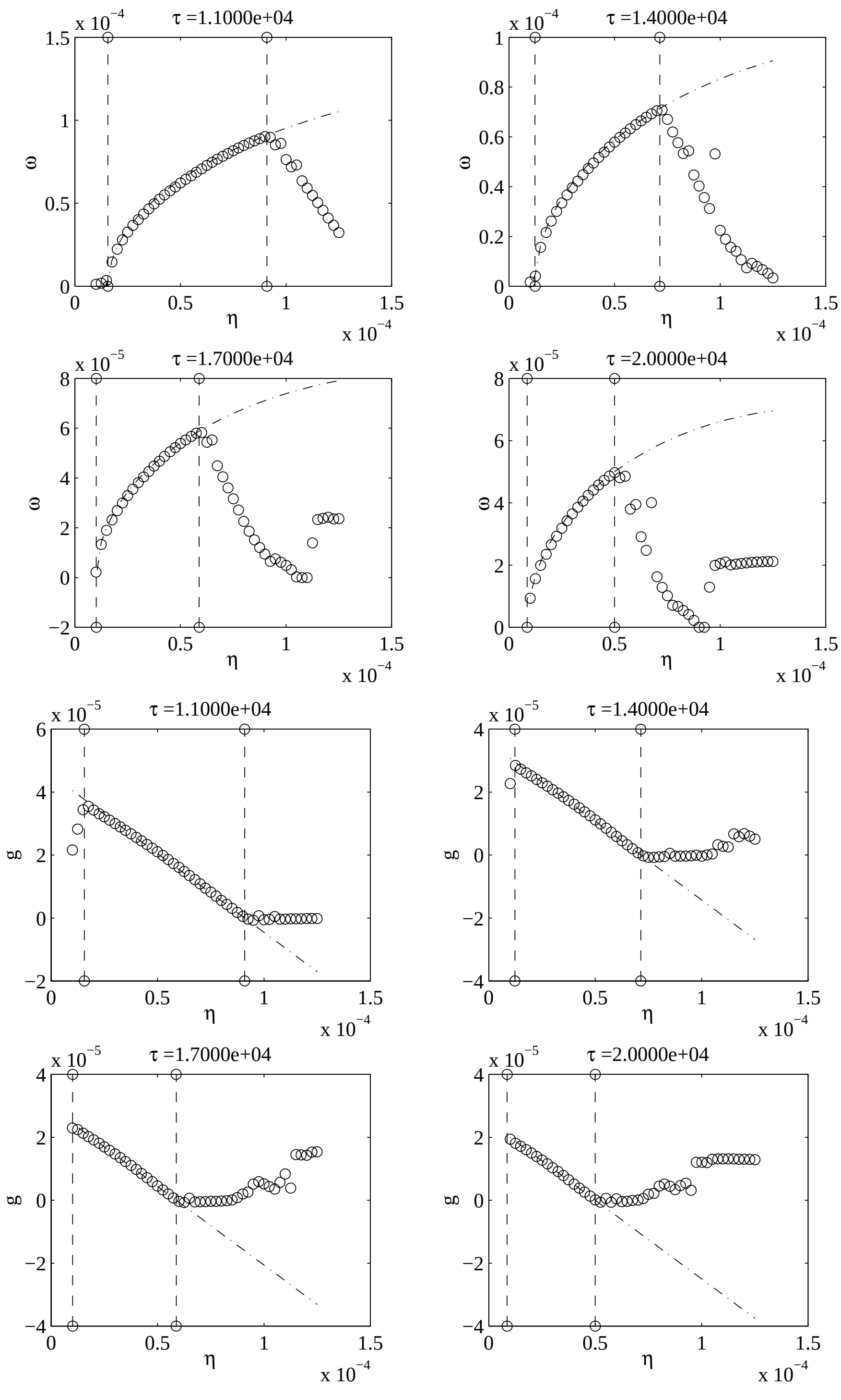




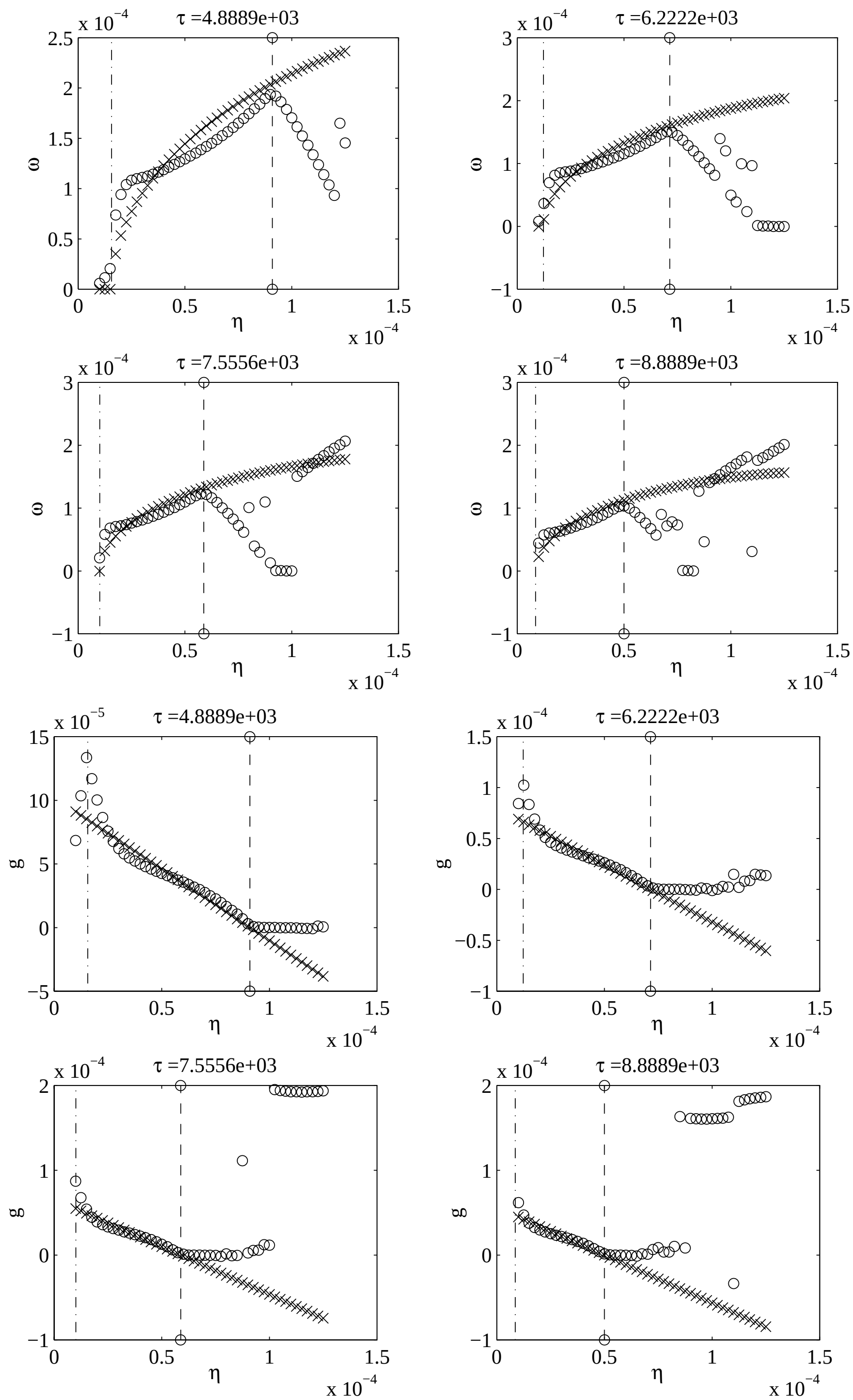
ω and g: $d = 0.5$



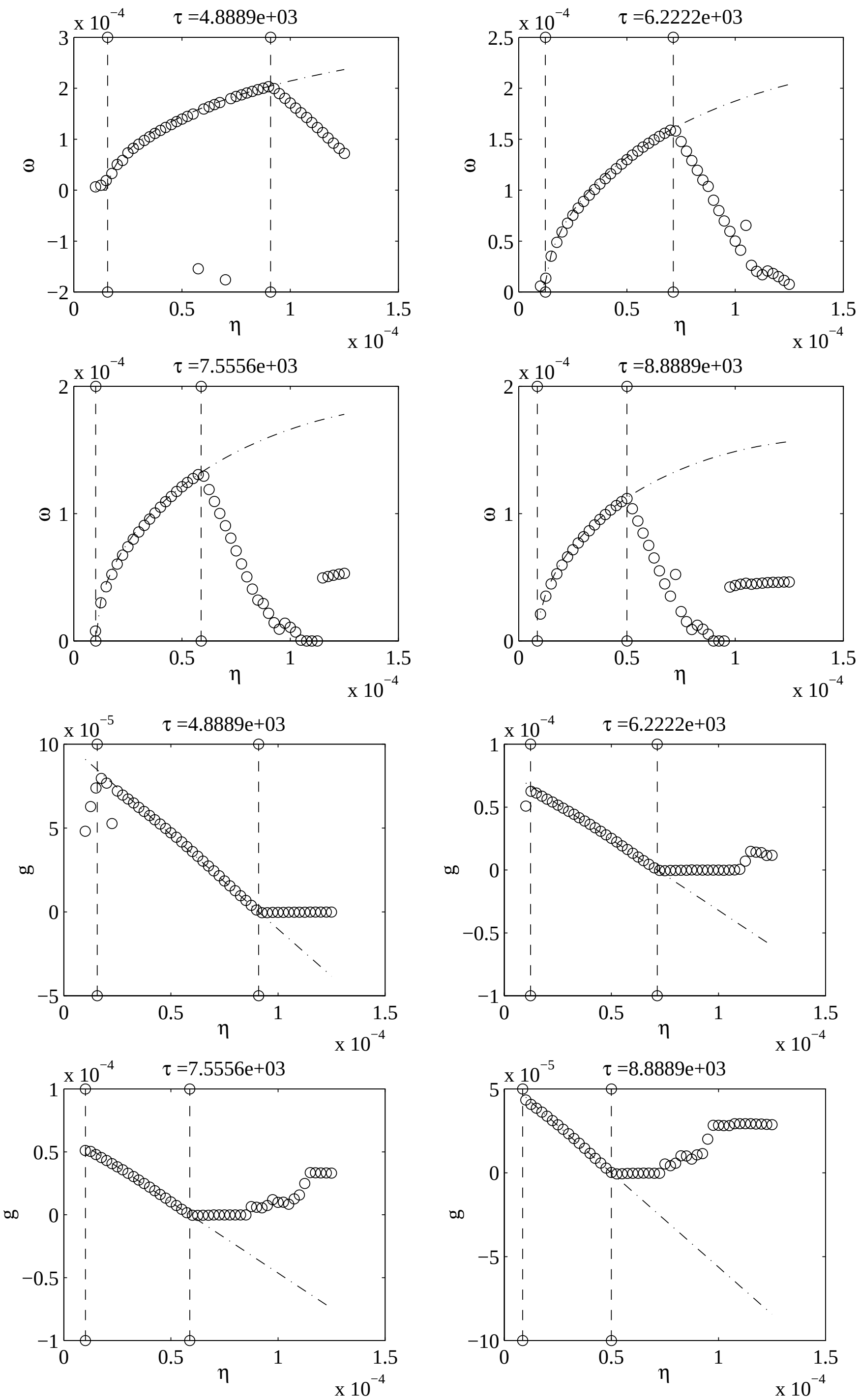
ω and g: $d = 1.0$



ω and g: $d = 1.5$



ω and g: $d = 1.5$



Conclusions

- Found a solution for simple sinusoidal oscillations
  - Very restricitve in parameter space
  - Had qualitative ω v.s. η prediction
- With some approximations, found a solution for damped oscillations
  - Robust with changes in parameters
  - Split parameter space into regions of validity
  - Quantitative prediction of the convergence of neuron
  - Identified some simulation-analysis differences