

Synaptic Modification in Spiking-Rate Models

A Comparison between Learning in
Spiking Neurons and Rate-Based Neuron Models

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Abstract

Rate-based neuron models have been successful in understanding many aspects of development such as the development of orientation selectivity(Bienenstock et al., 1982; Oja, 1982; Linsker, 1986; Miller, 1992; Bell and Sejnowski, 1997), the particular dynamics of visual deprivation(Blais et al., 1999) and the development of direction selectivity(Wimbauer et al., 1997; Blais et al., 2000). These models do not address phenomena such as temporal coding, spike-timing dependant synaptic plasticity, or any short-time behavior of neurons. More detailed spiking models (Song et.al, 2000; Shouval et.al. 2002; Yeung et.al. 2004) address these issues, and have had some success, but have failed to develop receptive fields in natural environments. These more detailed models are difficult to explore, given their large number of parameters and the run-time computational limitations. In addition, their results are often difficult to compare directly with the rate-based models.

We propose a model, which we call a spiking-rate model, which can serve as a middle-ground between the over simplistic rate-based models, and the more detailed spiking models. The spiking-rate model is a spiking model where all of the underlying processes are continuous Poisson, the summation of inputs is entirely linear (although non-linearities can be added), and the generation of outputs is done by calculating a rate output and then generating an appropriate Poisson spike train. In this way, the limiting behavior is identical to a rate-based model, but the properties of spiking models can be incorporated more easily. We present the development of receptive fields with this model in various visual environments. We then present the necessary conditions for the receptive field development in the spiking-rate models, and make comparisons to detailed spiking models, in order to more clearly understand the necessary conditions for receptive field development.

The Big Picture

- Rate-based models are successful at explaining diverse phenomena, such as
 - visual deprivation dynamics
 - direction selectivity
 - orientation and ocular dominance organizationwithin complex visual environments such as *natural images*
- Spiking models are successful at explaining
 - spike-timing dependent plasticity
 - LTP/LTD dynamics
 - selectivity in simplified environments.
- Spiking models have *not been successful* in more complex environments, such as natural images

Proposal

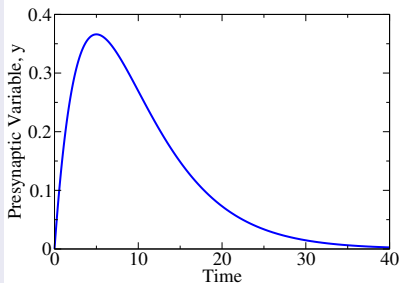
Introduce a simplified spiking model with closer analogues to the rate-based model

Spiking Rate Model: Introduction

- Can be used in place of integrate and fire
- Set of input spikes \rightarrow post-synaptic variable proportional to the input rate

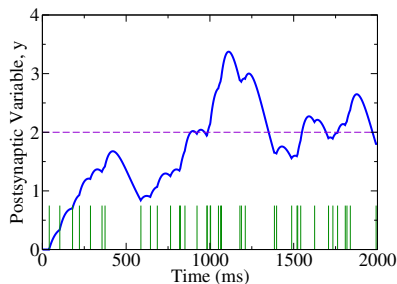
For each spike add...

Response Function for One Input Spike



Output converges to input rate.

2 Hz Input



Spiking Rate Model: Derivation

Consider 1 spike on 1 input fiber

- Define the desired postsynaptic variable, y .
- Introduce a dummy variable, v , with no physical significance.
- When a spike comes in at time, t_j , the dummy variable, v , is increased by a given amplitude, a , and then allowed to decay with a time constant, τ .

$$v(t) = a \cdot e^{-(t-t_j)/\tau} \text{ for } t > t_j$$

- The postsynaptic variable, y , is an integrated version of this dummy variable, v

$$\frac{dy}{dt} = \frac{1}{\tau}(v - y)$$

which is the same as

$$y = \frac{1}{\tau} \int_{-\infty}^t v(t') e^{-(t-t')/\tau} dt' = \frac{a}{\tau} t \cdot e^{-t/\tau} \text{ (single spike at } t = 0)$$

Spiking Rate Model: Multiple Inputs and Weights

- Each input has a dummy variable, v_i and weight, w_i
- For each spike at time t_j

$$v_i(t) = a \cdot e^{-(t-t_j)/\tau} \text{ for } t > t_j$$

- The result is summed over all spikes for a given input
- The postsynaptic variable is calculated from the sum

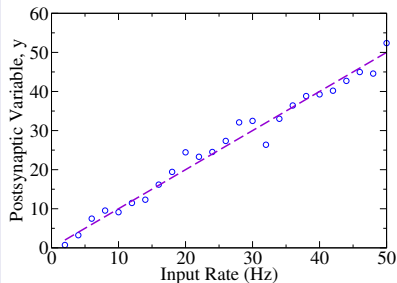
$$v = \sum_i v_i \cdot w_i$$
$$\frac{dy}{dt} = \frac{1}{\tau}(v - y)$$

- Use y , truncated for $y < 0$, as the rate of a poisson process

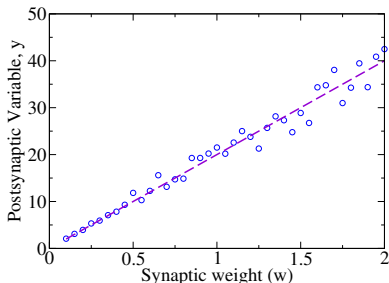
Spiking Rate Model: Linear Properties

- For a single input and $a \times \tau = 1000\text{ms}$ the postsynaptic variable, y , converges to the input rate, x
- The postsynaptic variable, y , is **linear** in
 - parameters: a and τ
 - architecture and environment: the synaptic weight, w , the input rate, x , the number of inputs

Input Rate

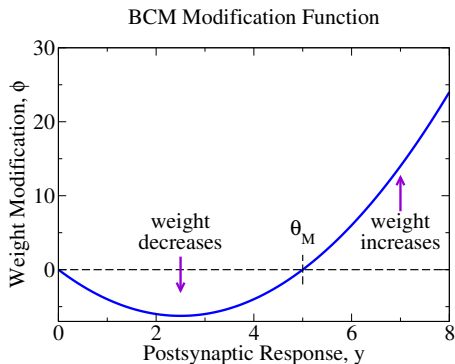


Synaptic weight



Rate-Based Model

BCM Synaptic Modification



- $y > \theta_M$: weight increases
- $y < \theta_M$: weight decreases
- θ_M : slides with response

$$\frac{dw_i}{dt} = \phi(y, \theta_M)x_i$$
$$\theta_M \sim E[y^2]$$

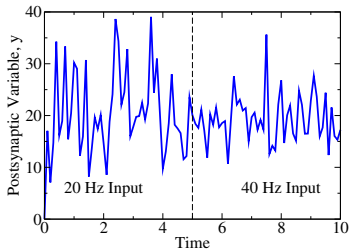
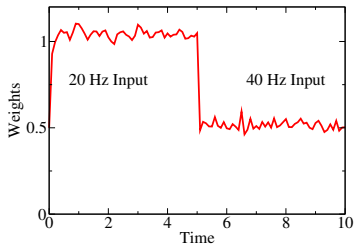
- Achieves selectivity and stability
- Can account for deprivation dynamics
- Properties:
 - Fixed point weight proportional to $1/\text{input} \Rightarrow y \sim \text{constant}$
 - $\theta_M \propto 1/\text{probability of selected pattern}$

Spiking-Based BCM Model

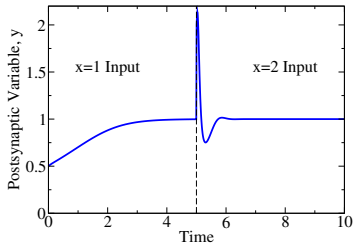
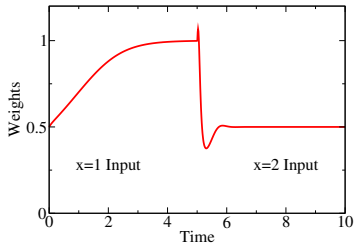
- For each connection, define
 - a presynaptic variable, x
 - a postsynaptic variable, ydefined as in the spiking-rate model
- Calculate the threshold, θ_M , from the postsynaptic variable
- Modify the weight based on the BCM modification equations
- All simulations for a single cell

1 Dimensional Comparison

- Spiking Model. $w_{\text{final}} \propto 1/x$ and $y \sim \text{constant}$

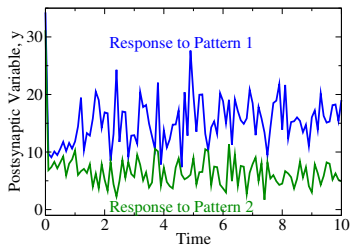
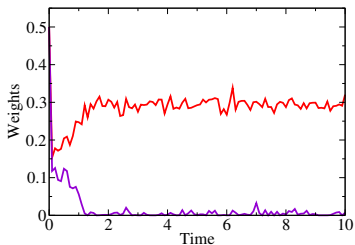


- Rate Model

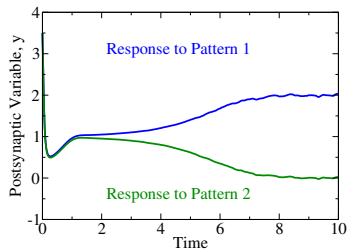
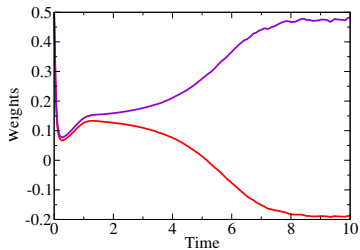


2 Dimensional Comparison

- Spiking Model. Selective to 1 of the 2 patterns.

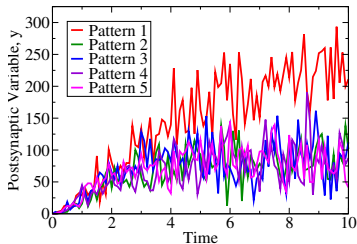
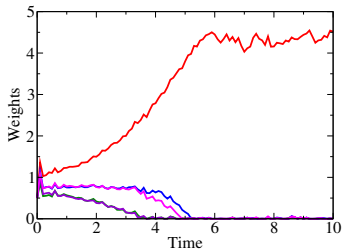


- Rate Model

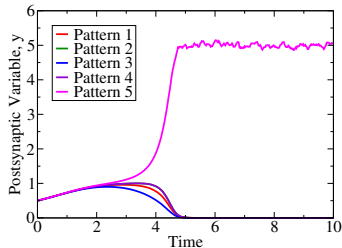
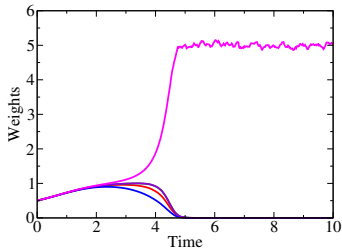


N Orthogonal Patterns Comparison

- Spiking Model. Selective to 1 of the N patterns.

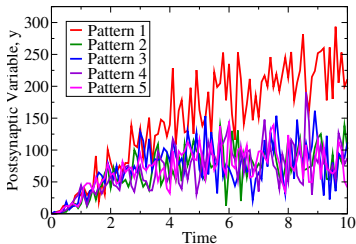
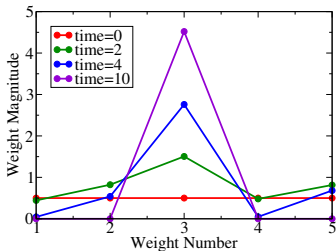


- Rate Model

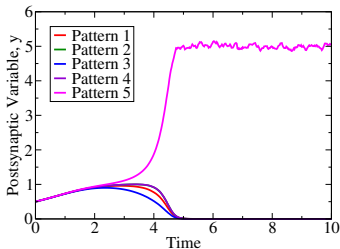
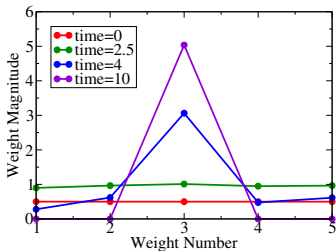


N Orthogonal Patterns Comparison

- Spiking Model. Selective to 1 of the N patterns.

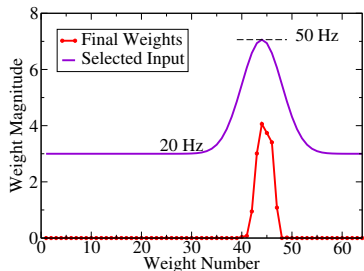


- Rate Model

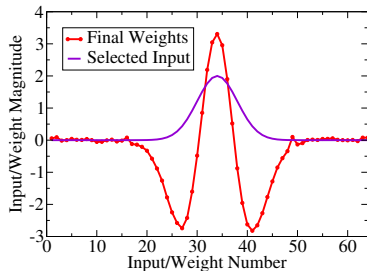


Gaussian Patterns Comparison

Final Weights From Spiking Model

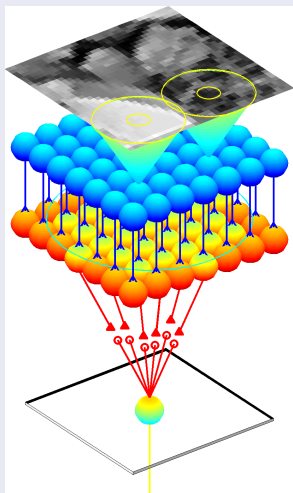


Final Weights From Rate-Based Model



Natural Images

Architecture

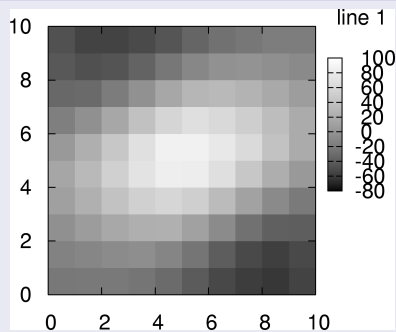


- Patches from Natural Images Environment
- Pre-processed with a Difference of Gaussians filter
- Sample patches:

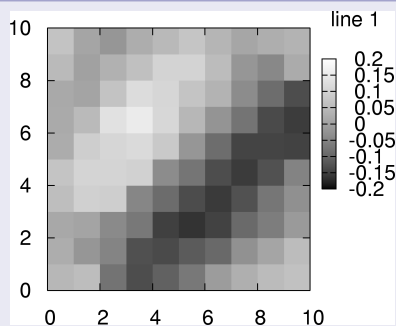


Natural Images: Rate versus Spiking

Receptive Field From Spiking Model



Receptive Field From Rate-Based Model



Software Packages

plasticity: web.bryant.edu/~bblais/plasticity



roctagon: web.bryant.edu/~bblais/octave



soctagon: web.bryant.edu/~bblais/octave



Conclusions

- Can reproduce many of the rate-based properties in a simple spiking model
- The comparison between the rate-based models to the spiking-rate model is transparent
- Will help bridge the gap between spiking and rate models

Further Work

- Instead of a rate variable, look at other properties of spike trains
 - temporal correlations
 - latencies
- Find a scenario for selectivity in natural image environment, with all-positive weights

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