Teaching a Spherical Chicken to Read

Feature Extraction in Neural Models

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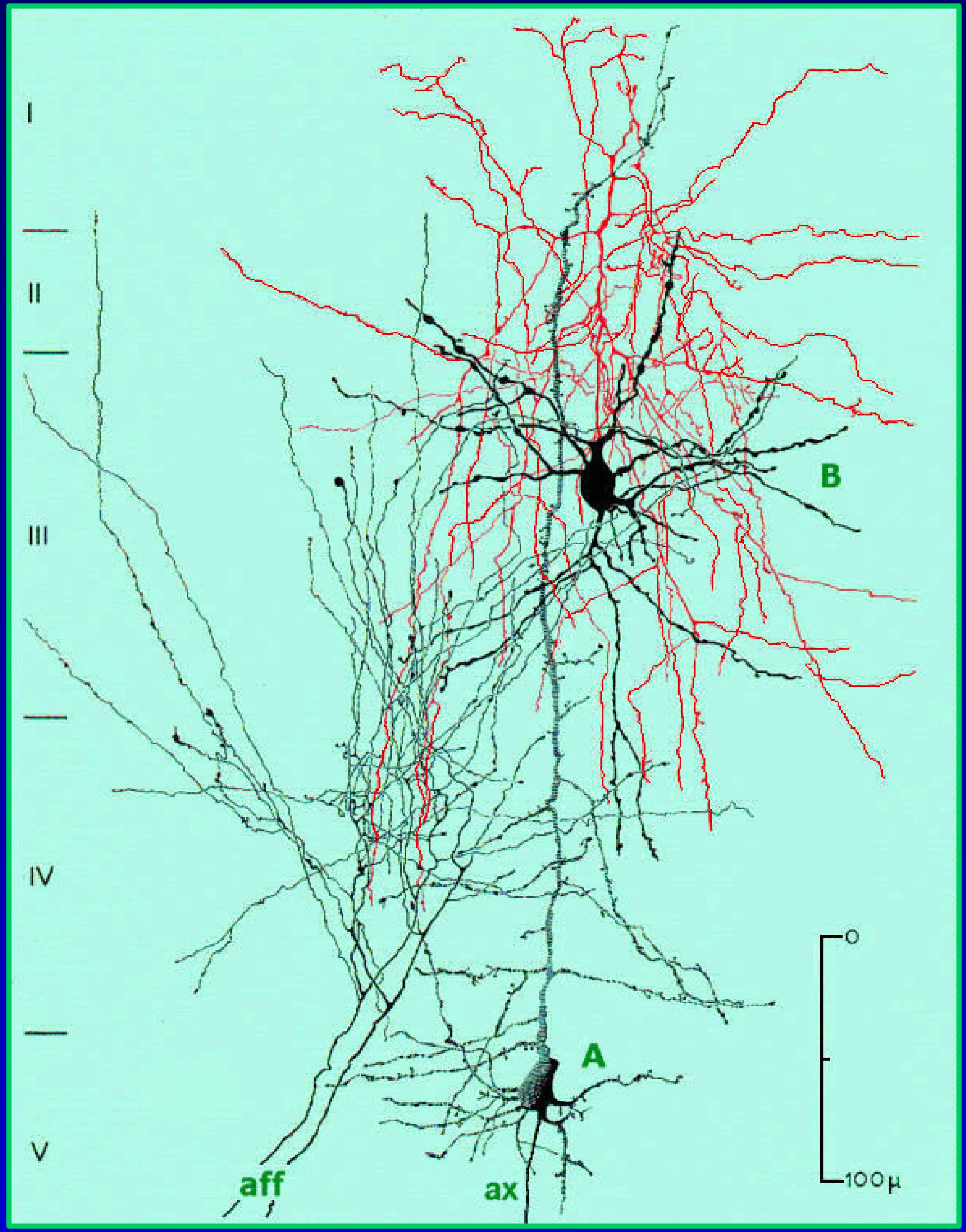
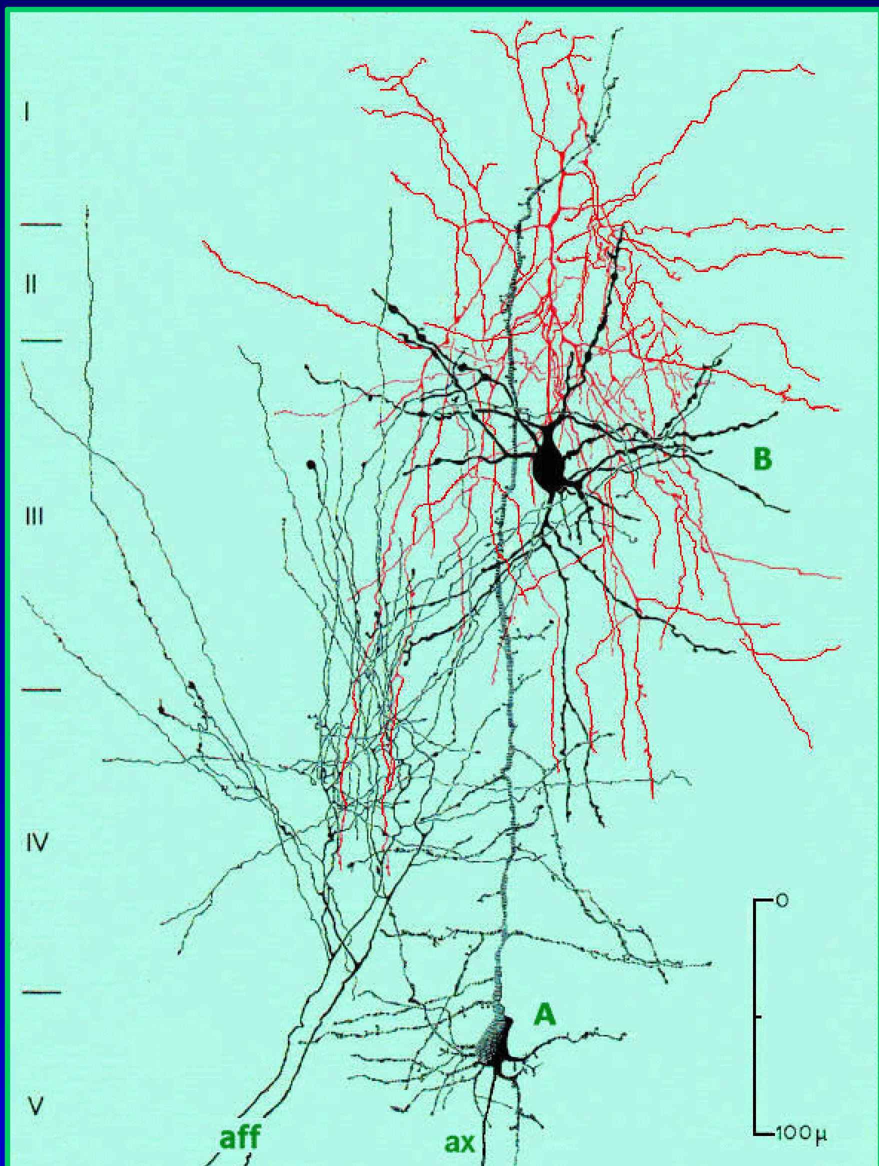
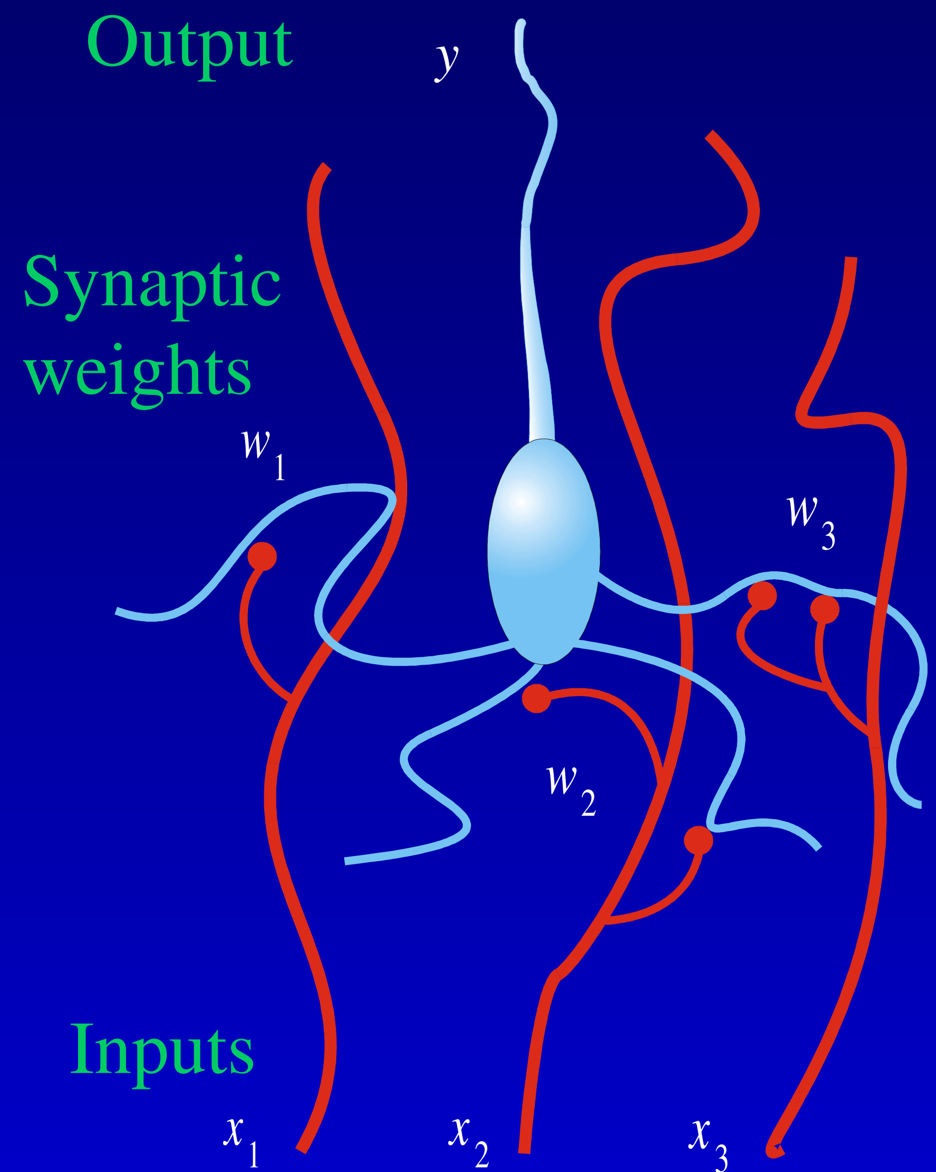
Science and Technology

A Neuron



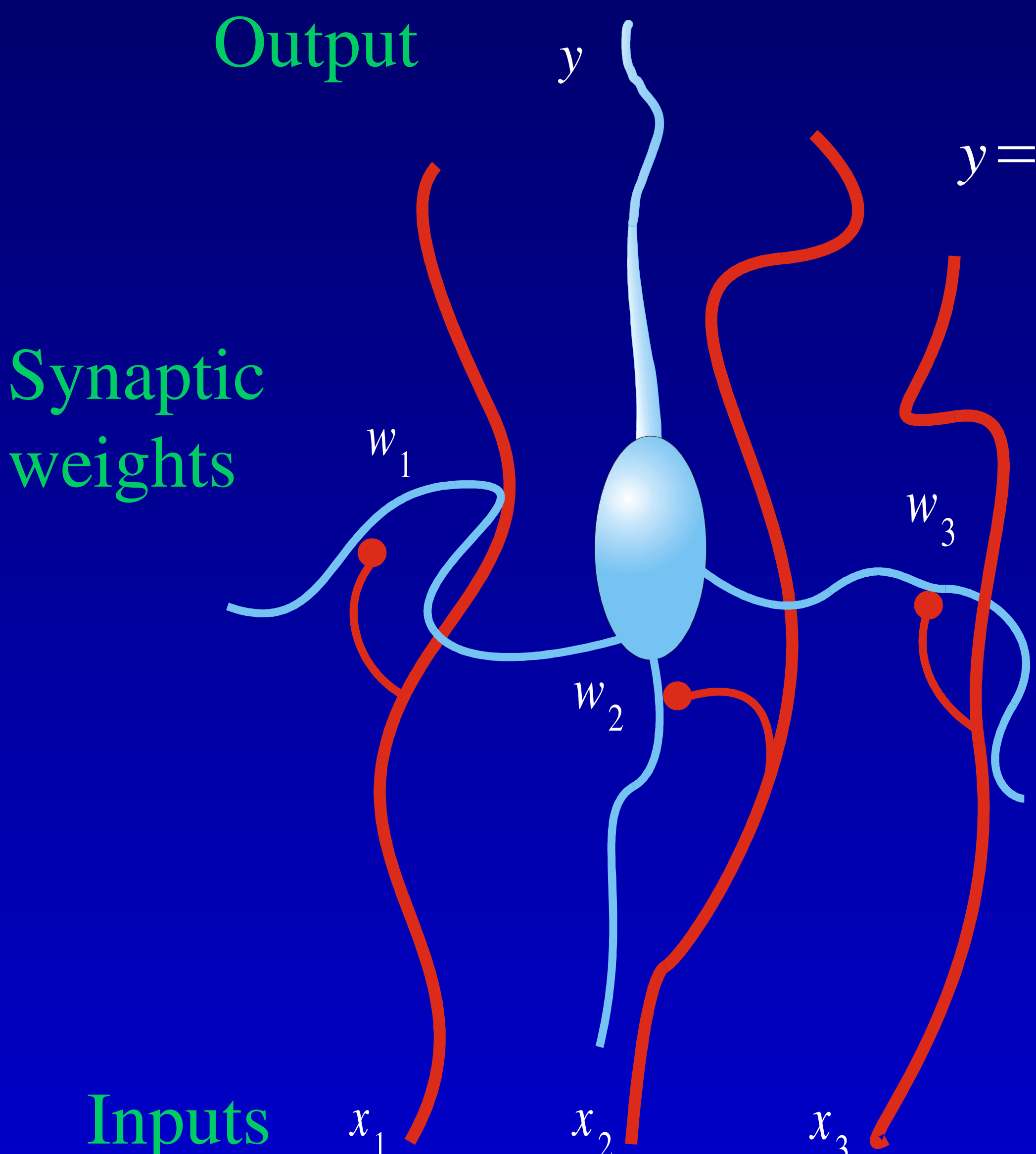
Simple Model of a Neuron

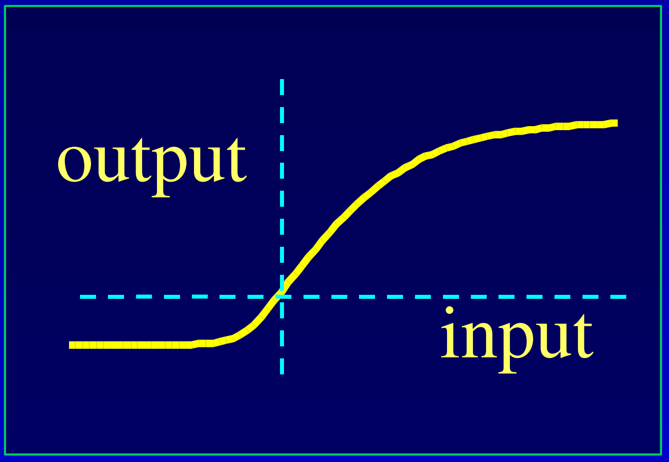




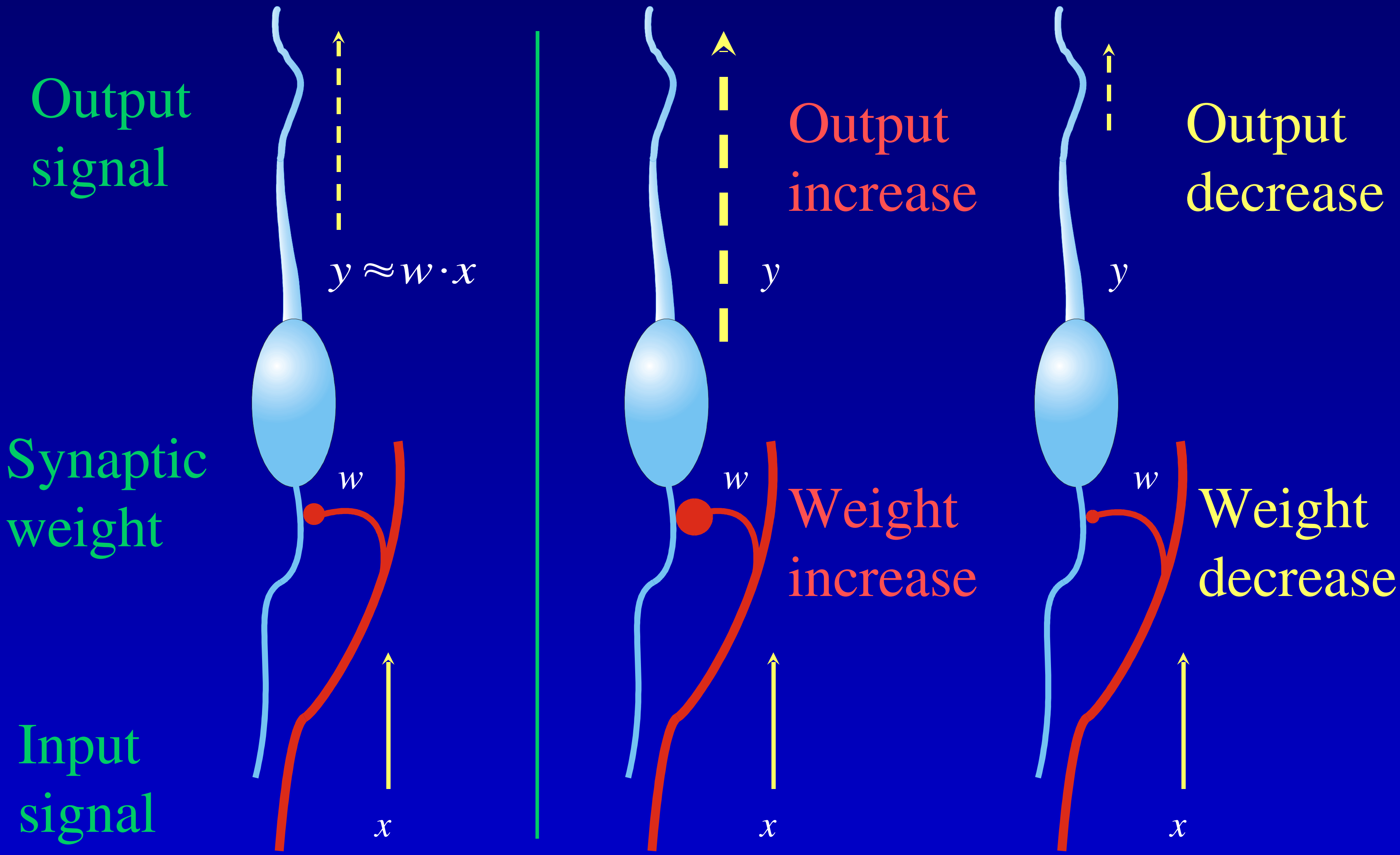
Neuron Output

$$y = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3 + \cdots$$





Synaptic Modification



Hebbian Learning

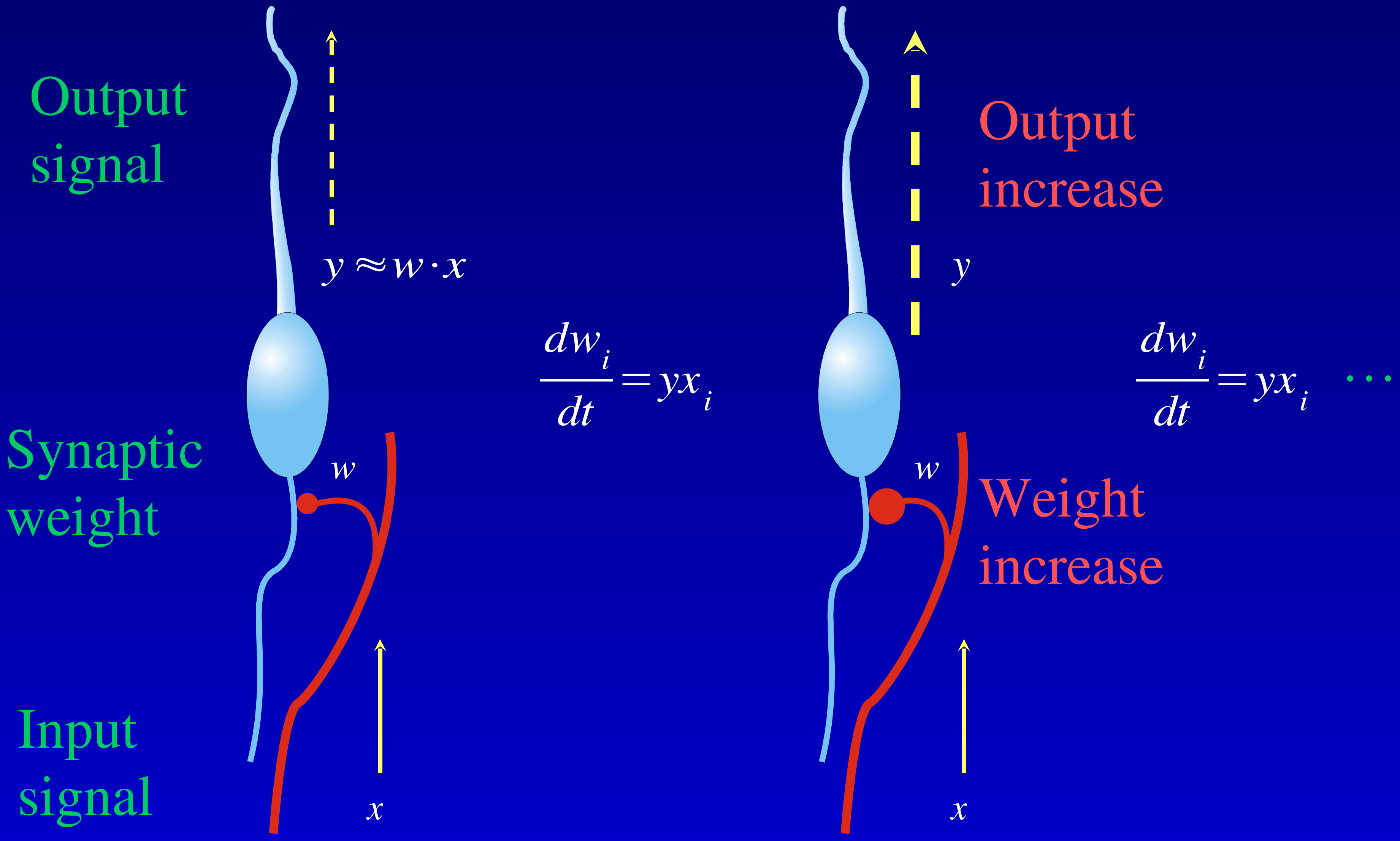
"When an axon in cell A is near enough to excite cell B and repeatedly and persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency in firing B is increased." - Hebb, 1949

"Those that fire together wire together"

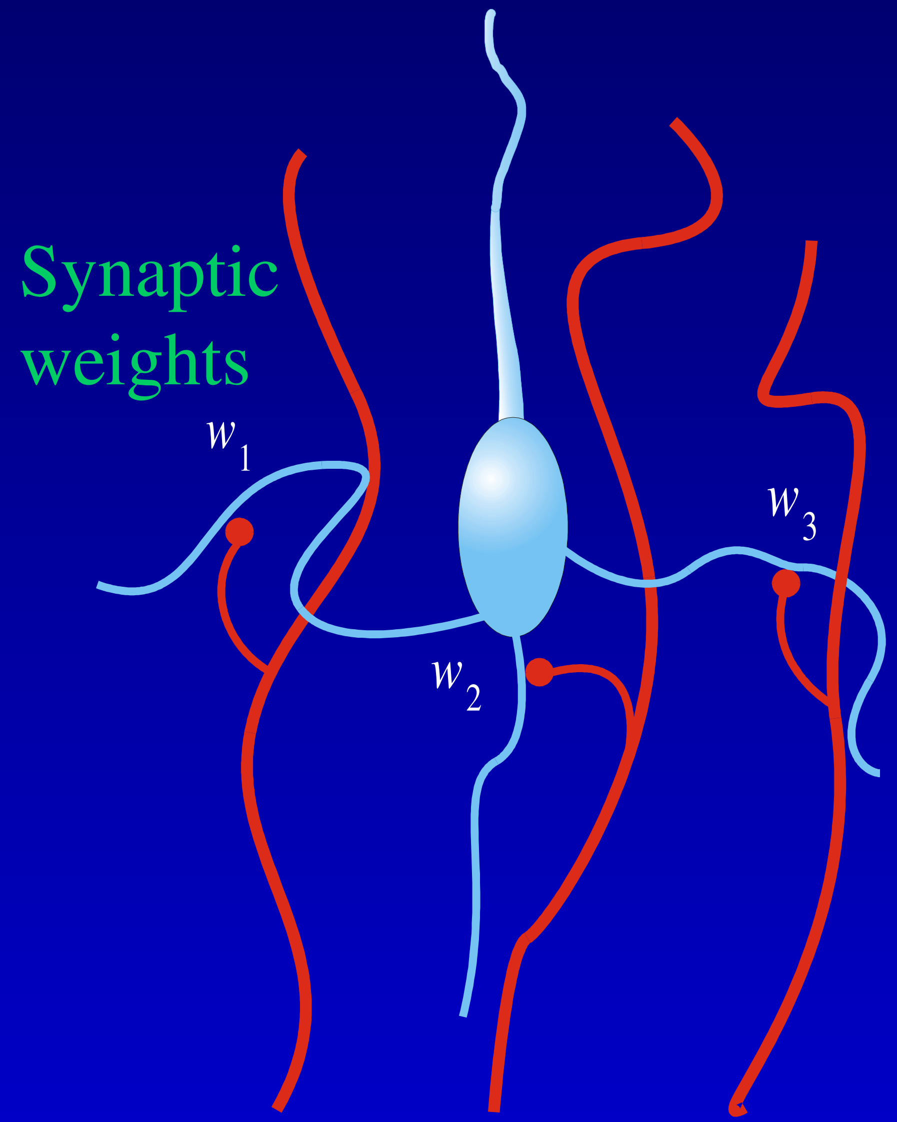
- Mathematically:

$$\frac{dw_i}{dt} = yx_i$$

Instability



Synaptic Stabilization



Mathematical method implies Biological mechanism

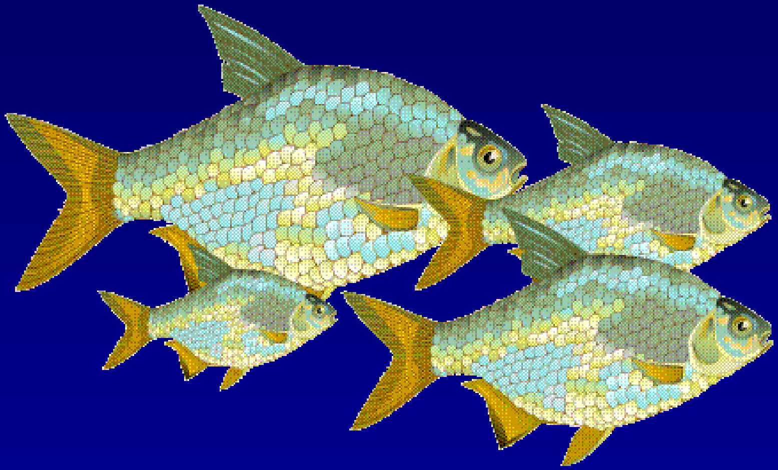
- Saturation limits $w_{min} \leq w_i \leq w_{max}$
- Normalization $\sum_i w_i^2 = \text{constant}$

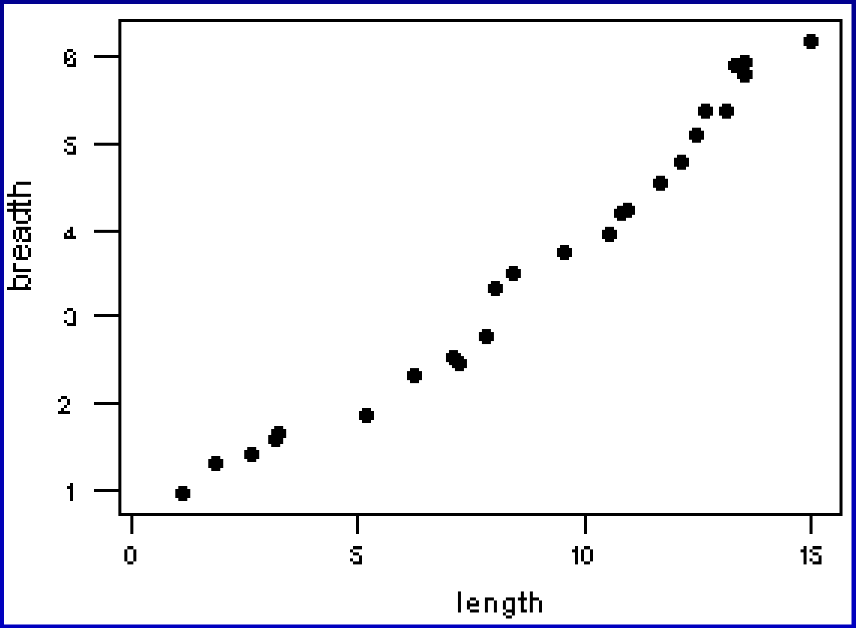
  (Linsker 1986;Miller 1994)
- Decay terms $\frac{dw_i}{dt} = yx_i - y^2 w_i$

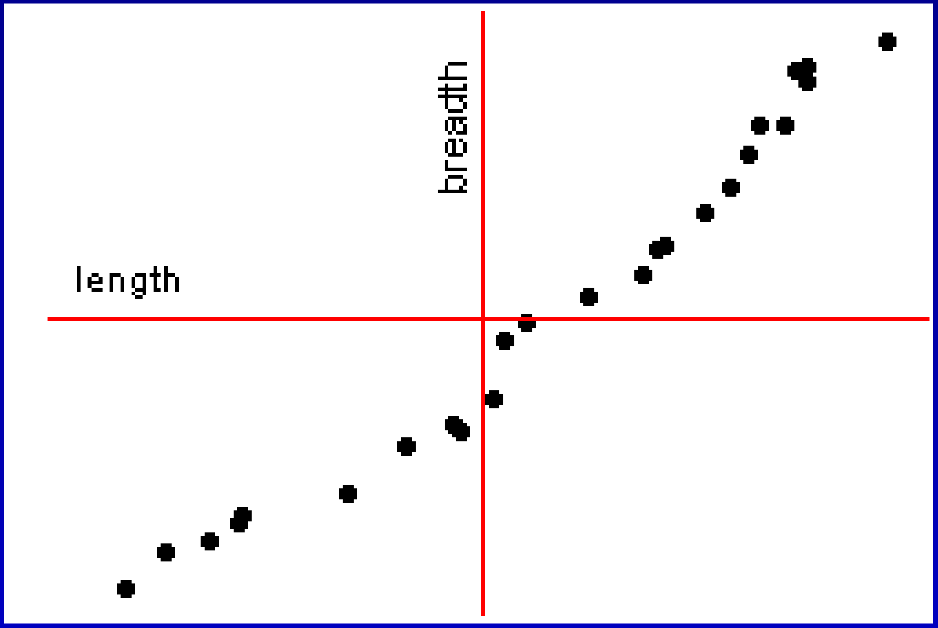
  (Oja 1982, Blais et. al. 1998)
- Moving threshold

  (BCM 1982, Blais et. al. 1999)

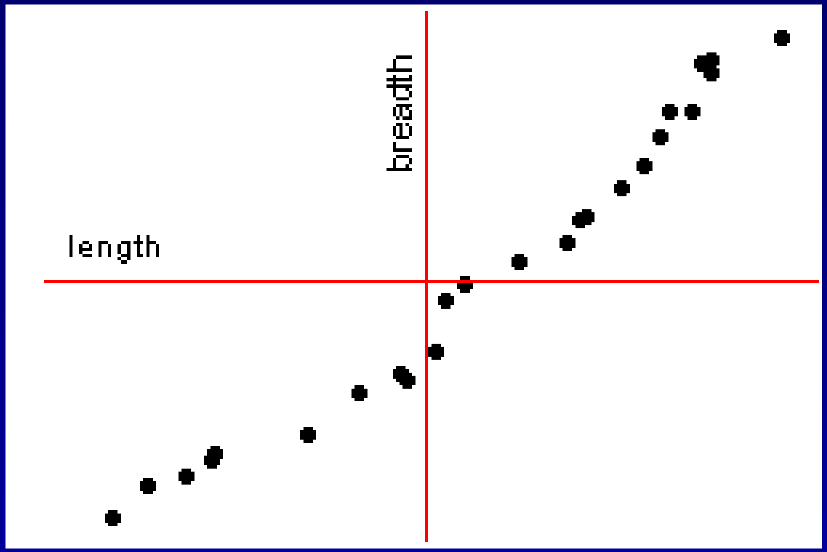
Feature Detection Example

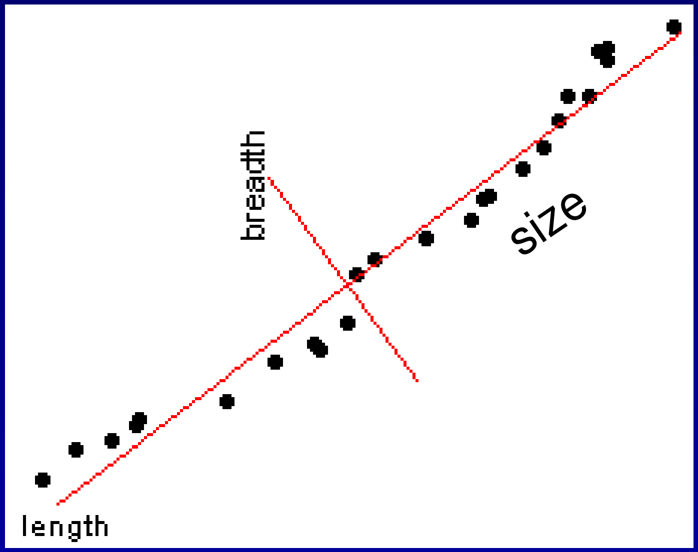






Feature Detection Example





$$¿¿\,w_1 \cdot (\text{length}) + w_2 \cdot (\text{breadth})$$

- Direction of highest variance

What Do Hebbian Neurons Do?

- Hebbian (Oja)
  - Choose *w* to find direction of high variance in *y*

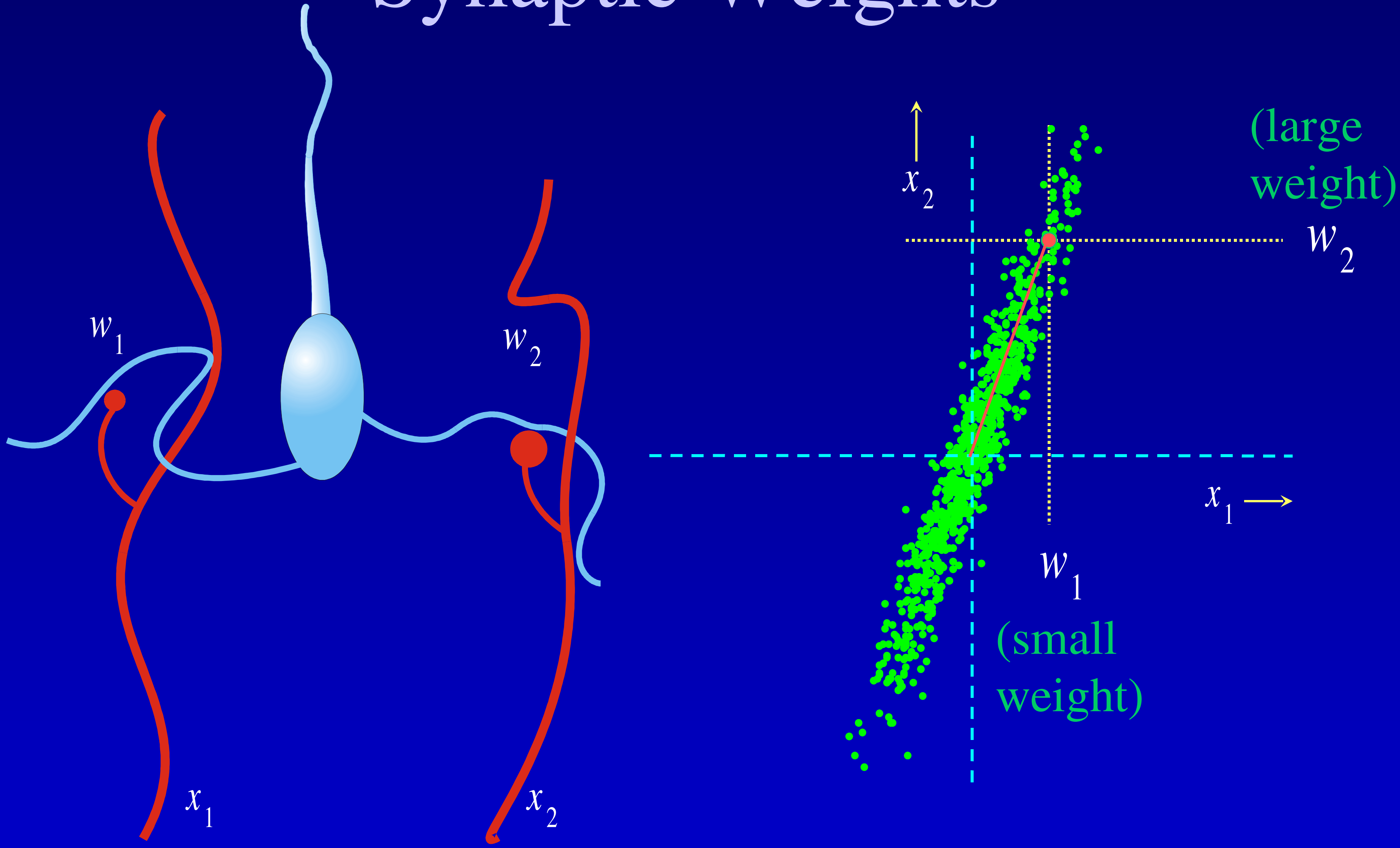
$$y = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3 + \cdots$$

$$V = \frac{1}{2} E\left[y^2\right]$$

Choose *w* to maximize variance: $$\frac{dV}{dw_i} = E\left[yx_i\right]$$

Hebbian modification: $$\frac{dw_i}{dt} = yx_i$$

Principle Components and Synaptic Weights



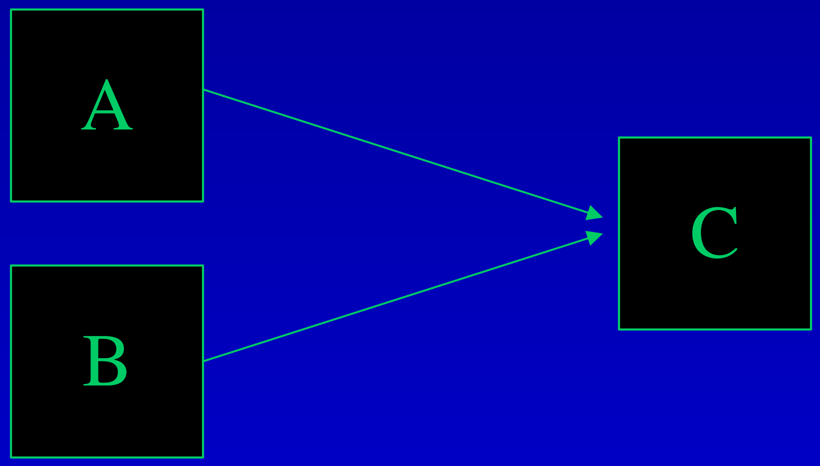
Non-Gaussianity

Projection Pursuit

$$y = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3 + \cdots$$

- Most choices of "*w*" are uninteresting.
- Interesting directions are non-Gaussian

Independent Component Analysis



- Mixture is more Gaussian than originals if A and B are independent
- Search for unmixing matrix so that unmixed signals are non-Gaussian

What Do These Neurons Do?

- Hebbian (Oja)

$$V = \frac{1}{2} E\left[y^2\right]$$

  - Direction of high variance

- BCM (Bienenstock, Cooper, Munro)

$$R = \frac{1}{3} E\left[y^3\right] - \frac{1}{4} E^2\left[y^2\right]$$

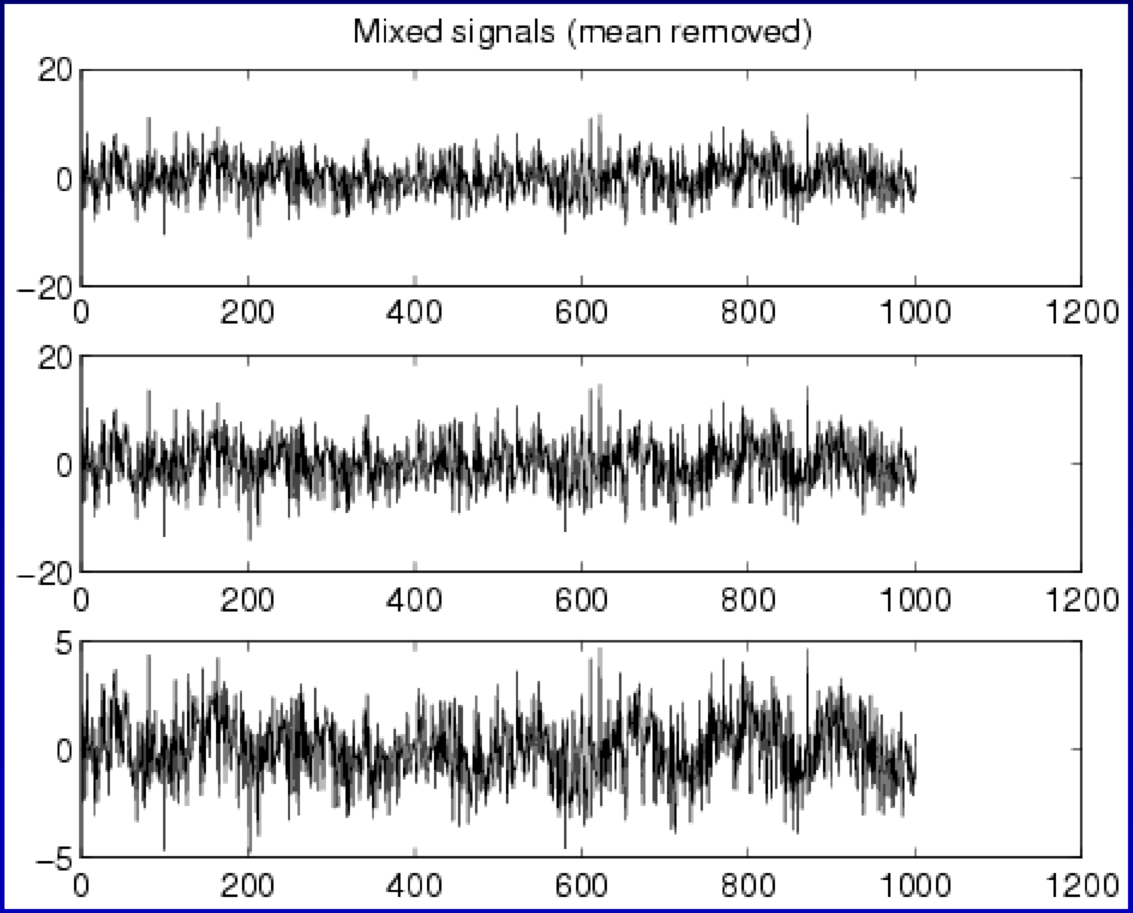
  - Finds directions with multi-modal projections
  - Clusters

- Kurtosis (Hyvarinen)

$$K = E\left[y^4\right] - 3\, E^2\left[y^2\right]$$

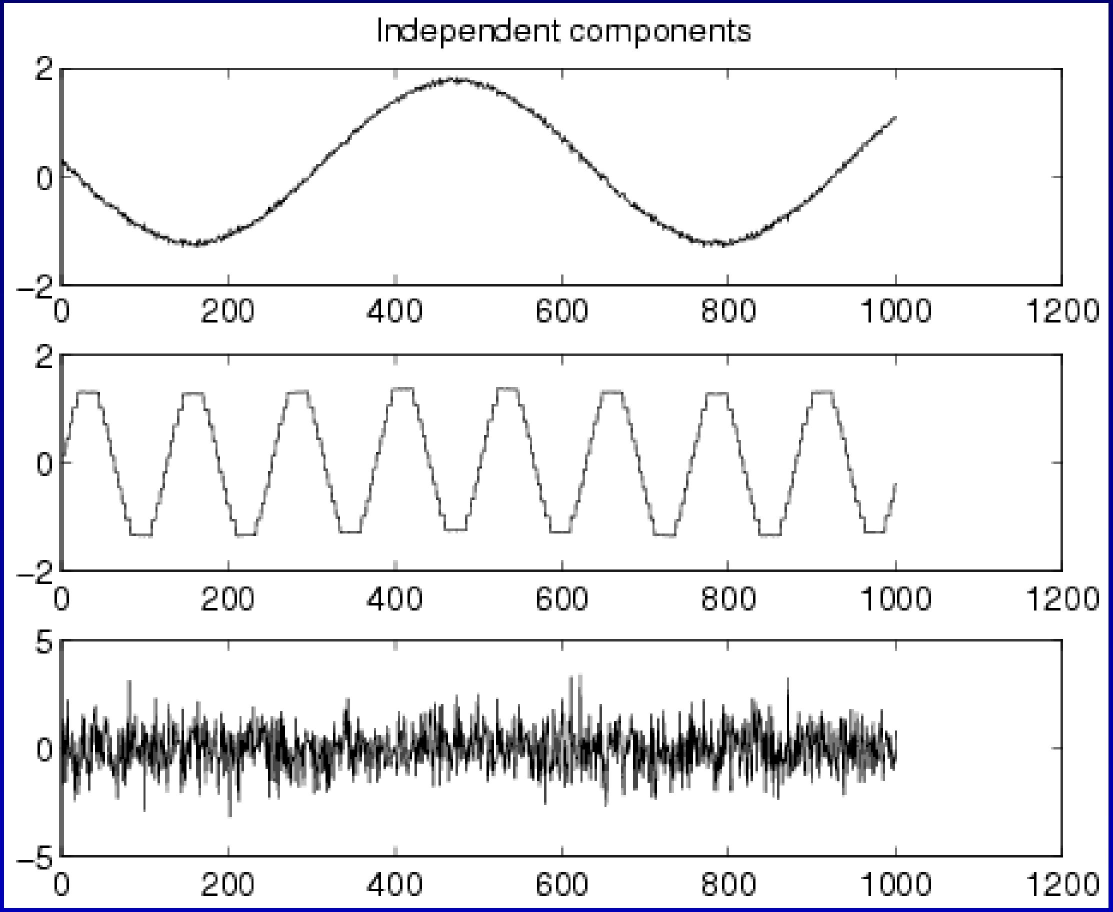
  - Finds directions with projections of high kurtosis
  - Blind source separation

Blind Source Separation



- Pure sine wave
- Step sine wave
- Gaussian noise

Blind Source Separation



- Pure sine wave
- Step sine wave
- Gaussian noise

Summary

- Start with a simple model of a neuron
  - Ignores complexities
  - Output is a weighted sum of inputs
- Learning = modifying the weights in the sum
- Mathematically identical to finding interesting features
- Finding weights which maximize some property of output statistics
  - Maximum variance — principal components
  - Multi-modality — clusters
  - Kurtosis — independent components

    unmixing