Time Dependence of Visual Deprivation: A Comparison between Models Of Plasticity and Experimental Results

Outline

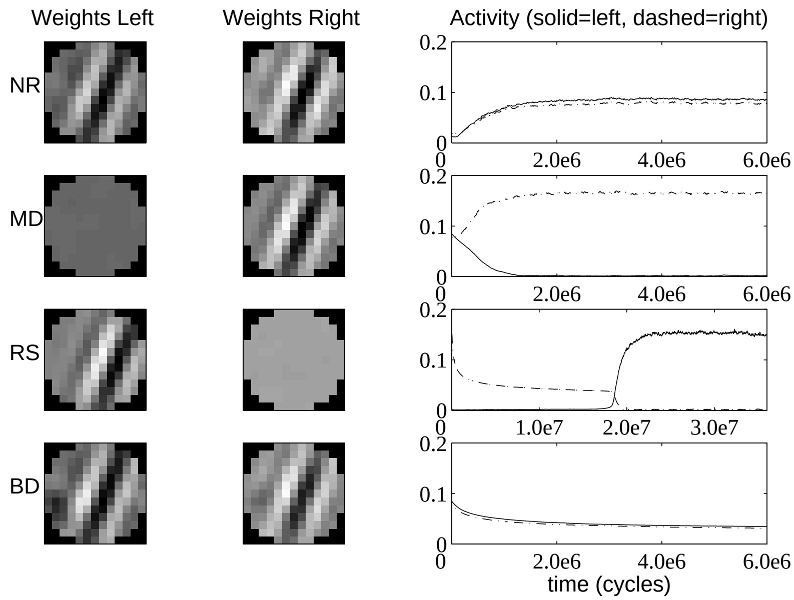
- Introduction to Experimental Results
- Introduction to the Models
  - · BCM
  - · PCA
- Results from Simulations
- PCA results from Wyatt Solution
- Conclusions

Classical Rearing Experiments

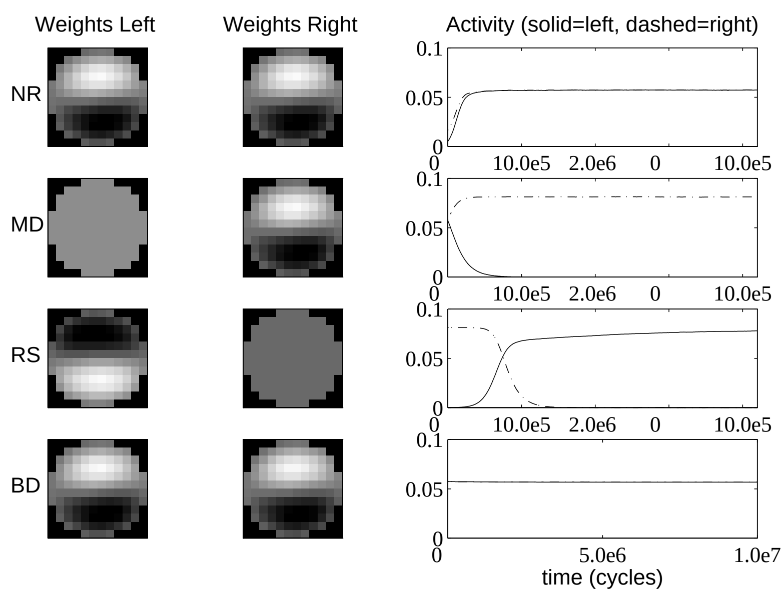
- Normal Rearing (NR):
  - · start **unselective** (random weights)
  - · **both** eyes are presented with input
  - ∗ develop **orientation selectivity** in **both** eyes
- Monocular Deprivation (MD):
  - · start after NR, with both eyes selective
  - · **one** eye is closed (presented with uncorrelated input)
  - ∗ closed eye **loses selectivity** and drops to zero activity
  - ∗ open eye becomes **stronger** (occular dominance shift)
- Binocular Deprivation (BD):
  - · start after NR, with both eyes selective
  - · **both** eyes are closed
  - ∗ **no** OD shift, neuron remains **binocular**
  - ∗ closed eyes drop in activity, but **not** to zero
  - ∗ drop in activity is **slower** than MD
- Reversed Suture (RS):
  - · start after MD, with one eye selective
  - · the closed eye is now open, and the open eye is closed
  - ∗ newly open eye recovers activity **after** the newly closed eye loses activity
  - ∗ drop in activity is **slower** than MD

Examples of the Models

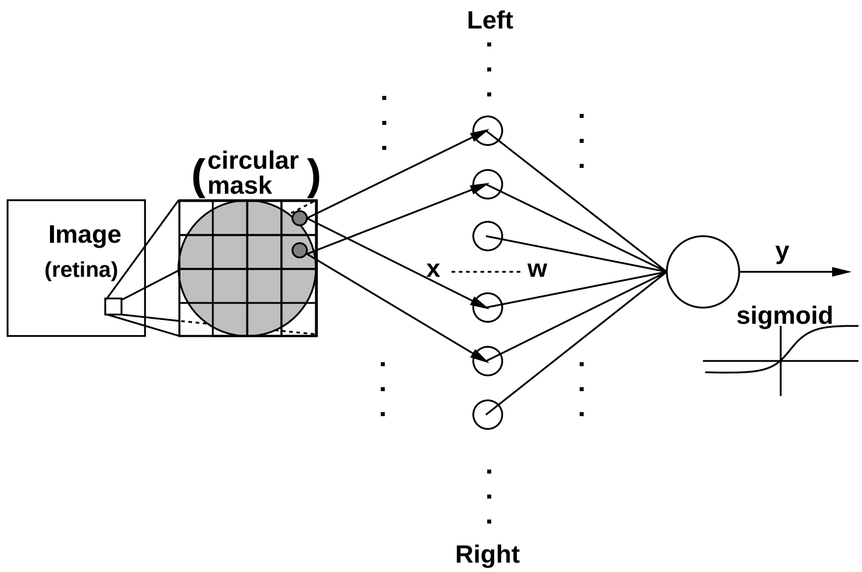
• BCM



• PCA



Single Cell with Natural Scene Input

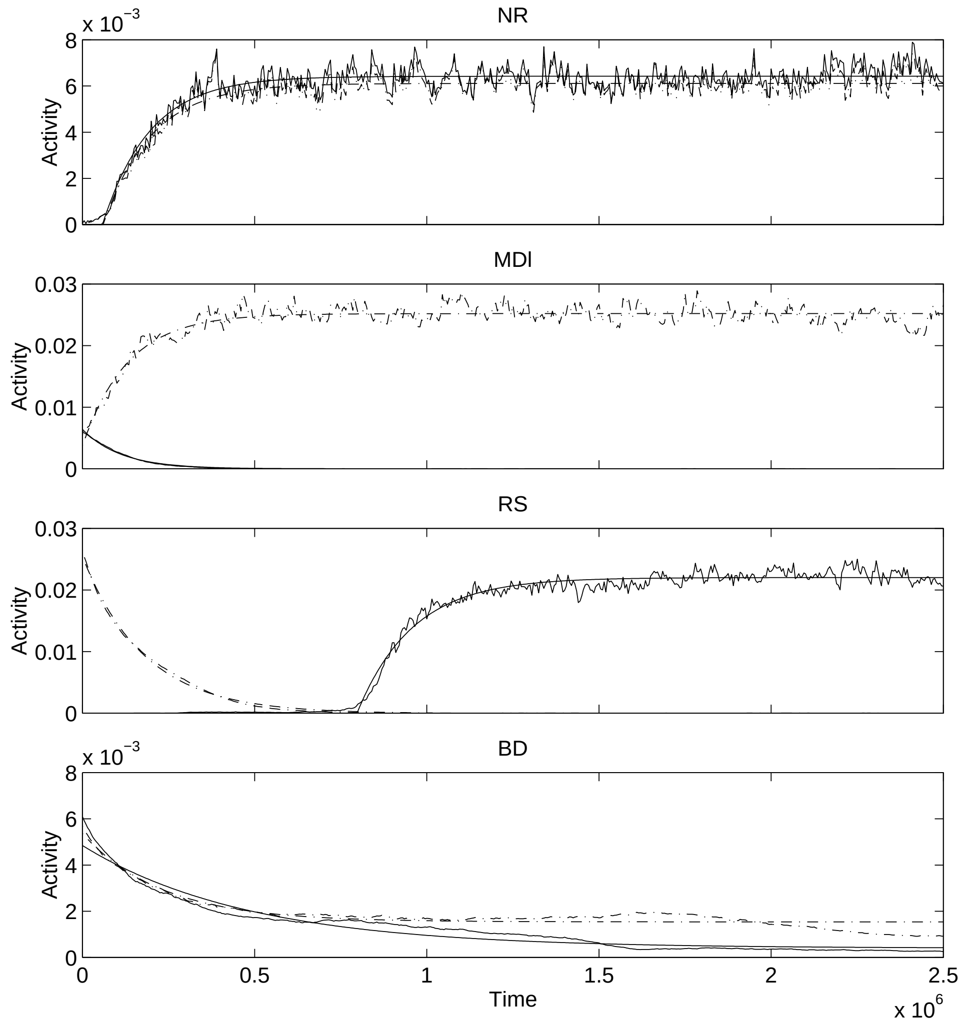


- image values : $\approx [-5 : 5]$
- sigmoid defined:

$$\sigma(x, v_1, v_2) \equiv \begin{cases} v_2\left(\frac{2}{1+\exp(-2x/v_2)} - 1\right) & \text{if } x > 0 \\ v_1\left(\frac{2}{1+\exp(-2x/v_1)} - 1\right) & \text{if } x < 0 \end{cases}$$

- sigmoid: $\sigma_{\text{cortical}}(x, -1.0, 50.0)$
- $y_{\text{BCM}} = \sigma_{\text{cortical}}(\mathbf{w} \cdot \mathbf{x})$, $y_{\text{PCA}} = \mathbf{w} \cdot \mathbf{x}$
- $\dot{\mathbf{w}}_{\text{BCM}} = \eta y(y - \theta)\mathbf{x}$, $\dot{\theta} = \frac{1}{\tau}(y^2 - \theta)$
- $\dot{\mathbf{w}}_{\text{PCA}} = \eta y(\mathbf{x} - y\mathbf{w})$

Calculating Deprivation Times



- fit to $Y(t) = H(t - t_0) \cdot \left(Y_1 + Y_0 e^{(t-t_0)/t_1}\right)$
- $H(t - t_0)$ is a (Heaviside) step function

Results

	PCA		BCM	
Ratio of $1/e$ times	mean	std	mean	std
$t_1^{\mathrm{NR}}/t_1^{\mathrm{MD}}$	1.36	1.08	.614	.225
$t_1^{\mathrm{BD}}/t_1^{\mathrm{MD}}$	1.97e6	5.71e6	4.97	1.45
$t_1^{\mathrm{RS}}/t_1^{\mathrm{MD}}$	2.07	.848	1.94	.505

Development $1/e$ Time Ratios for PCA and BCM.

- Average is taken over the parameter space
  - $\cdot\ \eta = 1 \cdot 10^{-6} - 1 \cdot 10^{-5}$
  - $\cdot\ \tau = 500 - 2900$ (for BCM only)

PCA: Wyatt Solution

- Wyatt PCA Solution:

$$\mathbf{w}(t) = \frac{e^{\mathbf{C}t}\mathbf{w}_o}{\left(\left\|e^{\mathbf{C}t}\mathbf{w}_o\right\|^2 + 1 - \|\mathbf{w}_o\|^2\right)^{1/2}}$$

$$\mathbf{C} \equiv \left\langle \mathbf{x}\mathbf{x}^{\mathrm{T}} \right\rangle, \langle \mathbf{x} \rangle = 0$$

- Binocular Deprivation:

$$\mathbf{C} = \begin{pmatrix} \sigma^2 & & \cdots & 0 \\ & \sigma^2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \sigma^2 \end{pmatrix}$$

$$\|\mathbf{w}_o\| = 1 \text{ (from Normal Rearing)}$$

$$\mathbf{w}(t) = \frac{e^{\sigma^2 t}\mathbf{w}_o}{e^{\sigma^2 t}} = \mathbf{w}_o$$

$$\Rightarrow \quad \text{Output does a random walk}$$

PCA: Wyatt Solution

- Normal Rearing:

$$\mathbf{w}(t) = \frac{e^{\mathcal{C}t}\mathbf{w}_o}{\left(\|e^{\mathcal{C}t}\mathbf{w}_o\|^2 + 1 - \|\mathbf{w}_o\|^2\right)^{1/2}}$$

$$\mathcal{C} = \left\langle \begin{pmatrix} \mathbf{x} \\ \mathbf{x} \end{pmatrix} \left(\mathbf{x}^{\mathrm{T}}\mathbf{x}^{\mathrm{T}}\right) \right\rangle = \begin{pmatrix} \mathbf{C} & \mathbf{C} \\ \mathbf{C} & \mathbf{C} \end{pmatrix}$$

· Approximation: $\lambda_1 \gg \lambda_2, \lambda_3, \ldots$

$$\mathbf{w}(t) \approx \frac{(e^{\lambda_1 t} - 1)(a_1^{\mathrm{l}} + a_1^{\mathrm{r}}) \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{pmatrix} + \mathbf{w}_o}{((e^{2\lambda_1 t} - e^{\lambda_1 t})(a_1^{\mathrm{l}} + a_1^{\mathrm{r}})^2 + 1)^{1/2}}$$

· development time depends critically on λ_1

· as $t \to \infty$

$$w(t) \to \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{pmatrix}$$

PCA: Wyatt Solution

- Normal Rearing (detailed):

$$\begin{aligned}
\mathbf{w}(t) &= \frac{e^{\mathcal{C}t}\mathbf{w}_o}{\left(\left\|e^{\mathcal{C}t}\mathbf{w}_o\right\|^2 + 1 - \left\|\mathbf{w}_o\right\|^2\right)^{1/2}} \\
\mathcal{C} &= \left\langle \begin{pmatrix} \mathbf{x} \\ \mathbf{x} \end{pmatrix} \left(\mathbf{x}^{\mathrm{T}}\mathbf{x}^{\mathrm{T}}\right) \right\rangle = \begin{pmatrix} \mathbf{C} & \mathbf{C} \\ \mathbf{C} & \mathbf{C} \end{pmatrix} \\
\mathbf{w}_o &= \begin{pmatrix} \mathbf{w}_o^{\mathrm{l}} \\ \mathbf{w}_o^{\mathrm{r}} \end{pmatrix} = \sum_j \begin{pmatrix} a_j^{\mathrm{l}}\mathbf{v}_j \\ a_j^{\mathrm{r}}\mathbf{v}_j \end{pmatrix} \\
\mathbf{C}\mathbf{v}_j &= \lambda_j \mathbf{v}_j
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}\mathbf{w}_o &= \begin{pmatrix} \mathbf{C} & \mathbf{C} \\ \mathbf{C} & \mathbf{C} \end{pmatrix} \sum_j \begin{pmatrix} a_j^{\mathrm{l}}\mathbf{v}_j \\ a_j^{\mathrm{r}}\mathbf{v}_j \end{pmatrix} = \sum_j \begin{pmatrix} \lambda_j \mathbf{v}_j (a_j^{\mathrm{l}} + a_j^{\mathrm{r}}) \\ \lambda_j \mathbf{v}_j (a_j^{\mathrm{l}} + a_j^{\mathrm{r}}) \end{pmatrix} \\
e^{\mathcal{C}t}\mathbf{w}_o &= \sum_j e^{\lambda_j t}(a_j^{\mathrm{l}} + a_j^{\mathrm{r}}) \begin{pmatrix} \mathbf{v}_j \\ \mathbf{v}_j \end{pmatrix} - \sum_j \begin{pmatrix} a_j^{\mathrm{r}}\mathbf{v}_j \\ a_j^{\mathrm{l}}\mathbf{v}_j \end{pmatrix}
\end{aligned}$$

· Approximation: $\lambda_1 \gg \lambda_2, \lambda_3, \ldots$

$$\mathbf{w}(t) \approx \frac{(e^{\lambda_1 t} - 1)(a_1^{\mathrm{l}} + a_1^{\mathrm{r}}) \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{pmatrix} + \mathbf{w}_o}{\left((e^{2\lambda_1 t} - e^{\lambda_1 t})(a_1^{\mathrm{l}} + a_1^{\mathrm{r}})^2 + 1\right)^{1/2}}$$

PCA: Wyatt Solution

- Monocular Deprivation:

$$\mathcal{C} = \left\langle \begin{pmatrix} \mathbf{x} \\ \mathbf{n} \end{pmatrix} \left(\mathbf{x}^{\mathrm{T}} \mathbf{n}^{\mathrm{T}} \right) \right\rangle = \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \end{pmatrix}$$

$$\mathbf{w}_o = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{pmatrix} \text{ (from NR)}$$

$$\mathbf{w}(t) \approx \frac{\begin{pmatrix} e^{\lambda_1 t} \mathbf{v}_1 \\ e^{\sigma^2 t} \mathbf{v}_1 \end{pmatrix}}{\left(e^{2\lambda_1 t} + e^{2\sigma^2 t} \right)^{1/2}}$$

· timing depends on relationship between λ_1 and σ^2

PCA: Wyatt Solution

- Monocular Deprivation (detailed):

$$\begin{aligned}
\mathcal{C} &= \left\langle \begin{pmatrix} \mathbf{x} \\ \mathbf{n} \end{pmatrix} \left(\mathbf{x}^{\mathrm{T}} \mathbf{n}^{\mathrm{T}}\right) \right\rangle = \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \end{pmatrix} \\
\mathbf{w}_o &= \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{pmatrix} \text{ (from NR)} \\
\mathbf{C}\mathbf{v}_j &= \lambda_j \mathbf{v}_j
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}\mathbf{w}_o &= \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \sigma^2 \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \lambda_1 \mathbf{v}_1 \\ \sigma^2 \mathbf{v}_1 \end{pmatrix} \\
e^{\mathcal{C}t}\mathbf{w}_o &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\lambda_1} \mathbf{v}_1 \\ e^{\sigma^2} \mathbf{v}_1 \end{pmatrix}
\end{aligned}$$

$$\mathbf{w}(t) \approx \frac{\begin{pmatrix} e^{\lambda_1 t} \mathbf{v}_1 \\ e^{\sigma^2 t} \mathbf{v}_1 \end{pmatrix}}{\left(e^{2\lambda_1 t} + e^{2\sigma^2 t}\right)^{1/2}}$$

PCA: Wyatt Solution

- Reversed Suture:

$$\mathcal{C} = \left\langle \begin{pmatrix} \mathbf{n} \\ \mathbf{x} \end{pmatrix} \left(\mathbf{n}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}} \right) \right\rangle = \begin{pmatrix} \sigma^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix}$$

$$\mathbf{w}_o = \begin{pmatrix} \mathbf{v}_1 \\ \epsilon \mathbf{v}_1 \end{pmatrix} \text{ (from MD)}$$

$$\mathbf{w}(t) \approx \frac{\begin{pmatrix} e^{\sigma^2 t} \mathbf{v}_1 \\ e^{\epsilon \lambda_1 t} \mathbf{v}_1 \end{pmatrix}}{\left(e^{2\sigma^2 t} + e^{2\epsilon \lambda_1 t} \right)^{1/2}}$$

· timing depends, again, on noise levels and the 1st eigenvalue

· there is no mechanism to keep the newly opened eye from recovering immediately

PCA: Wyatt Solution

- Reversed Suture (detailed):

$$\begin{aligned}
\mathcal{C} &= \left\langle \begin{pmatrix} \mathbf{n} \\ \mathbf{x} \end{pmatrix} \left(\mathbf{n}^{\mathrm{T}} \mathbf{x}^{\mathrm{T}} \right) \right\rangle = \begin{pmatrix} \sigma^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix} \\
\mathbf{w}_o &= \begin{pmatrix} \mathbf{v}_1 \\ \epsilon \mathbf{v}_1 \end{pmatrix} \text{ (from MD)} \\
\mathcal{C}\mathbf{w}_o &= \begin{pmatrix} \sigma^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{v}_1 \\ \epsilon \mathbf{v}_1 \end{pmatrix} = \begin{pmatrix} \sigma^2 \mathbf{v}_1 \\ \epsilon \lambda_1 \mathbf{v}_1 \end{pmatrix} \\
e^{\mathcal{C}t}\mathbf{w}_o &= \begin{pmatrix} e^{\sigma^2 t} \mathbf{v}_1 \\ e^{\epsilon \lambda_1 t} \mathbf{v}_1 \end{pmatrix}
\end{aligned}$$

$$\mathbf{w}(t) \approx \frac{\begin{pmatrix} e^{\sigma^2 t} \mathbf{v}_1 \\ e^{\epsilon \lambda_1 t} \mathbf{v}_1 \end{pmatrix}}{\left(e^{2\sigma^2 t} + e^{2\epsilon \lambda_1 t} \right)^{1/2}}$$

Conclusions

- BCM time ratios match more closely to biology than PCA time ratios
- PCA binocular deprivation performs a random walk about NR fixed point
- PCA monocular deprivation, reversed suture times depend critically on the largest eigenvalue of the covariance matrix
- PCA reversed suture has no quantity to keep the newly opened eye from recovering before the newly closed eye shuts off

In the Future

- Use the Wyatt solution to more fully understand PCA deprivation simulations
- Look at the effect of noise level on the times, perhaps drawing from biological experiments

Parameters

Parameter	Significance	Value
R_r, pixels	Diameter of the retinal patches	12
STD_c, pixels	SD of the ganglion excitatory-center Gaussian dist.	1.0
STD_s, pixels	SD of the ganglion inhibitory-surround Gaussian dist.	3.0
$\sigma(-\infty)$	lower limit on cortical sigmoid	-1.0
$\sigma(\infty)$	Upper limit on cortical sigmoid	50.0
$\bar{n^2}$, Hz^2	Average square of the noise from a closed eye	$0.333 : [-1 : 1]$
$\theta(0)$	Starting value of the BCM threshold	1.1
$m_i(0)$	Starting value of the synaptic weights for normal rearing	$0.0 - 0.1$
τ, iterations	Time constant in the definition of θ (BCM only)	$500 - 3000$
η, iterations^{-1}	Learning rate	$\approx$ 1e-6-1e-5